

Auctions with Endogenously Determined Biddable Combinations

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- It's Natural for Bidders to Bid on What They Value.
 - Anything else, creates risks, transaction costs, and perhaps, extra gaming opportunities
- For Spectrum, Bidder's Values are Greater for Combinations.
 - Exception: Budget Constraints
- So Far, FCC Auctions Have not Allowed Bids on Combinations.
 - Why Not?
 1. "Free rider" problem?
 2. Maximizing revenue **MIGHT** be hard Computationally.
 3. Game theorists came up with a really **CLEVER** partial solution.
 4. No time to try to do it right.
 5. Other?

- If Computability Risk is the Issue, There are Alternatives.
 - The “political solution” to the integer program
 - But math programmers are not as influential as game theorists?
 - The FCC could limit biddable combinations.
 - See (Rothkopf, Pekec, and Harstad 1998).
 - But the FCC would have to decide what is biddable.
 - No combinations is not a decision?
 - MTAs are not REQUIRED combos of BTAs?
 - **Let the Bidders Prioritize the Combinations.**
 - (This talk)

The Basic Idea

1. Bidders Can Bid on Individual Items.
 2. Bidders Prioritize Combinations and Can Bid on Them, Too.
 3. Combinations are Considered in Priority Order.
 - i.e., 1st priority combinations of all bidders (in addition to singleton bids), then 2nd priority combinations (in addition to 1st priority combos and singleton bids), then 3rd, and so on.
 4. Consideration of Combinations Ends When ..
 - a. All prioritized combinations have been considered, or
 - b. Computational time has been exhausted.
- Intended for One-Time Auctions, But Adaptable to Multi-Round Auctions.
 - Previous round winning combos would be 1st priority in the next round.

How is This Likely to Work?

- *Some A Priori Clues*

- Integer programming is better than it was.
- Worst-case bounds on computation are rarely close.
- Economic synergies tend to be systematic.

A Test Problem

- 153 Licenses
 - 27 out of 180 (from 12×15 matrix) are pre-owned by bidders

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1															
2			B					B					B		
3	A					A					A				
4					C					C					C
5															
6			B					B					B		
7	A					A					A				
8					C					C					C
9															
10			B					B					B		
11	A					A					A				
12					C					C					C

- 3 Serious Bidders
- Differing Synergistic Values

3 Bidders

- Different raw values
(in the following slide)
- Different synergies
 - A : High (0.4)
 - B : Medium (0.3)
 - C : Low (0.2)
- Qualitative information about ...
 - Other bidders' raw values
 - Other bidders' synergies

Raw Values of Bidders

A

A	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	7	5	4	4	5	7	5	4	4	5	7	5	4	3.4	3
2	13	7		5	7	13	7		5	7	13	7		4	3.4
3	21	13	7	7	13	21	13	7	7	13	21	13	7	5	4
4	13	7	5	4		13	7	5	4		13	7	5	4	
5	7	5	4	4	5	7	5	4	4	5	7	5	4	3.4	3
6	13	7		5	7	13	7		5	7	13	7		4	3.4
7	21	13	7	7	13	21	13	7	7	13	21	13	7	5	4
8	13	7	5	5		13	7	5	5		13	7	5	4	
9	7	5	4	4	5	7	5	4	4	5	7	5	4	3.4	3
10	13	7		5	7	13	7		5	7	13	7		4	3.4
11	21	13	7	7	13	21	13	7	7	13	21	13	7	5	4
12	13	7	5	5		13	7	5	5		13	7	5	4	

B

B	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	6	8	14	8	6	6	8	14	8	6	6	8	14	8	6
2	8	14	22	14	8	8	14	22	14	8	8	14	22	14	8
3		8	14	8	6		8	14	8	6		8	14	8	6
4	5	6	8	6		5	6	8	6		5	6	8	6	
5	6	8	14	8	6	6	8	14	8	6	6	8	14	8	6
6	8	14	22	14	8	8	14	22	14	8	8	14	22	14	8
7		8	14	8	6		8	14	8	6		8	14	8	6
8	5	6	8	6		5	6	8	6		5	6	8	6	
9	6	8	14	8	6	6	8	14	8	6	6	8	14	8	6
10	8	14	22	14	8	8	14	22	14	8	8	14	22	14	8
11		8	14	8	6		8	14	8	6		8	14	8	6
12	5	6	8	6		5	6	8	6		5	6	8	6	

C

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	5.7	6	6.4	7	8	7	6.4	6.4	7	8	7	6.4	6.4	7	8
2	6	6.4		8	10	8	7		8	10	8	7		8	10
3		7	8	10	16		8	8	10	16		8	8	10	16
4	7	8	10	16	24	16	10	10	16	24	16	10	10	16	24
5	6.4	7	8	10	16	10	8	6.4	10	16	10	8	6.4	10	16
6	6	6.4		8	10	8	7		8	10	8	7		8	10
7		7	8	10	16		8	8	10	16		8	8	10	16
8	7	8	10	16	24	16	10	10	16	24	16	10	10	16	24
9	6.4	7	8	10	16	10	8	6.4	10	16	10	8	6.4	10	16
10	6	6.4		8	10	8	7		8	10	8	7		8	10
11		7	8	10	16		8	8	10	16		8	8	10	16
12	7	8	10	16	24	16	10	10	16	24	16	10	10	16	24

Synergy Effects

- Bidder A's value of asset (7, 5)

$$= 13 + 0.4 \times (7 + 0 + 21 + 7)$$

Raw value of (7,5) A's synergy factor Value of (7,4) if it's allocated

(8,5) pre-owned by C (7,6) already owned (6,5) if allocated

A	4	5	6	7
6	5	7	13	7
7	7	13	21	13
8	5		13	7
9	4	5	7	5

Test Sets (So Far)

- Our own bidders
 - 1 case
 - Rothkopf vs. Rothkopf vs. Rothkopf
- FCC bidder advisors
 - 2 cases
 - 2 As vs. 1 B vs. 1 C
- Economics professors
 - (Still waiting for bids)
- All the combinations of above
 - 12 cases
 - 3 As vs. 2 Bs vs. 2 Cs

Four Intermediate Allocations

(-1)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	C	B	B	B	C	C	B	B	B	C	C	B	B	B	C
2	A	B	B	B	C	A	B	B	B	C	A	B	B	B	C
3	A	A	B	C	A	A	A	B	C	A	A	A	B	C	C
4	A	C	C	C	C	A	C	C	C	C	A	C	C	C	C
5	A	B	B	C	C	C	C	B	C	C	C	C	B	C	C
6	A	B	B	B	C	A	A	B	B	C	A	A	B	B	C
7	A	A	B	C	A	A	A	B	C	A	A	A	B	C	C
8	A	C	C	C	C	A	C	C	C	C	A	C	C	C	C
9	A	B	B	C	C	C	C	B	C	C	C	C	B	C	C
10	A	B	B	B	C	A	B	B	B	C	A	B	B	B	C
11	A	A	B	C	A	A	A	B	C	A	A	A	B	C	C
12	A	C	C	C	C	A	C	C	C	C	A	C	C	C	C

(1) Allocation with single bids

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	A	B	B	B	C	C	B	B	B	C	C	B	B	B	C
2	A	B	B	B	C	A	B	B	B	C	A	B	B	B	C
3	A	A	B	C	C	A	A	B	C	C	A	A	B	C	C
4	A	C	C	C	C	C	C	C	C	C	C	C	C	C	C
5	A	B	B	C	C	C	C	B	C	C	C	C	B	C	C
6	A	B	B	B	C	A	A	B	B	C	A	A	B	B	C
7	A	A	B	C	C	A	A	B	C	C	A	A	B	C	C
8	A	C	C	C	C	C	C	C	C	C	C	C	C	C	C
9	A	B	B	C	C	C	C	B	C	C	C	C	B	C	C
10	A	B	B	B	C	A	B	B	B	C	A	B	B	B	C
11	A	A	B	C	C	A	A	B	C	C	A	A	B	C	C
12	A	C	C	C	C	C	C	C	C	C	C	C	C	C	C

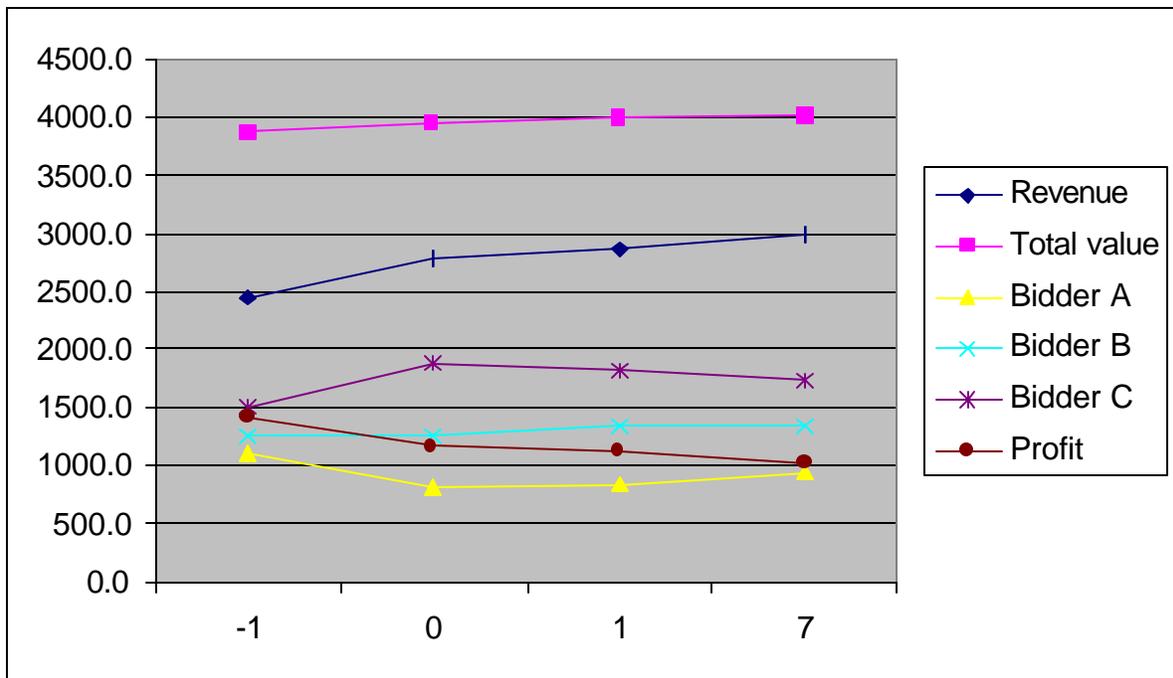
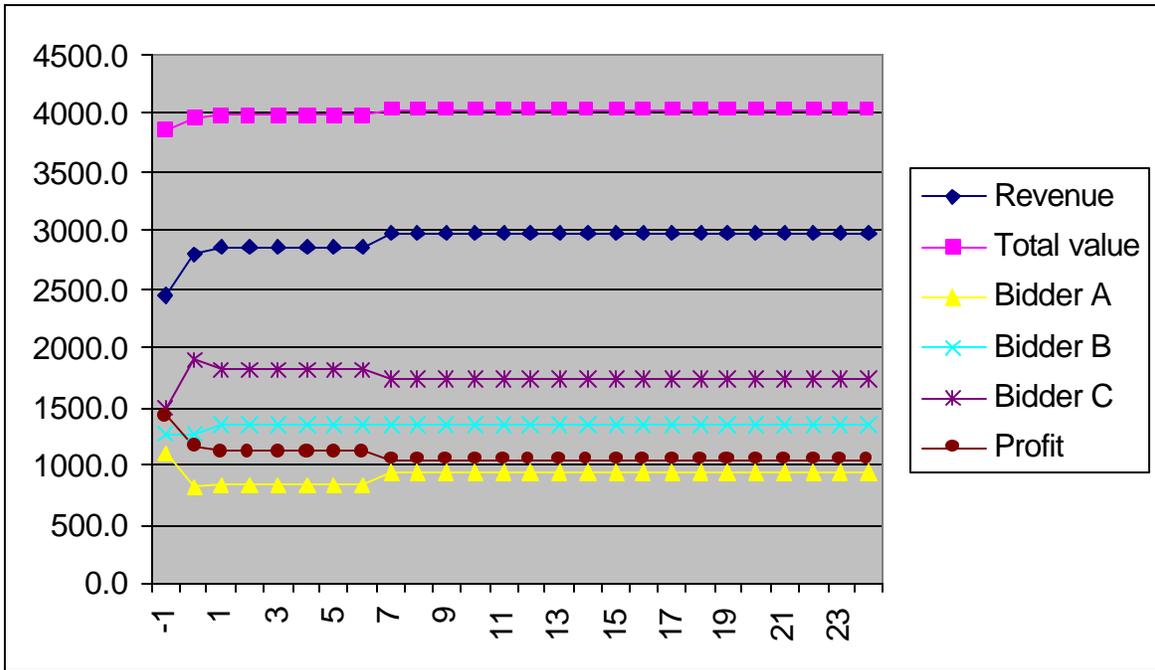
(2) Allocation with 1st priority list

-1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	A	B	B	B	C	A	B	B	B	C	A	B	B	B	C
2	A	B	B	B	C	A	B	B	B	C	A	B	B	B	C
3	A	A	B	C	C	A	A	B	C	C	A	A	B	C	C
4	A	C	C	C	C	C	C	C	C	C	C	C	C	C	C
5	A	B	B	C	C	C	B	B	C	C	C	B	B	C	C
6	A	B	B	B	C	A	A	B	B	C	A	A	B	B	C
7	A	A	B	C	C	A	A	B	C	C	A	A	B	C	C
8	A	C	C	C	C	C	C	C	C	C	C	C	C	C	C
9	A	B	B	C	C	C	B	B	C	C	C	B	B	C	C
10	A	B	B	B	C	A	B	B	B	C	A	B	B	B	C
11	A	A	B	C	C	A	A	B	C	C	A	A	B	C	C
12	A	C	C	C	C	C	C	C	C	C	C	C	C	C	C

(3) Allocation with 2nd priority list

-7	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	A	B	B	B	A	A	B	B	B	A	A	B	B	B	C
2	A	B	B	B	A	A	B	B	B	A	A	B	B	B	C
3	A	A	B	C	C	A	A	B	C	C	A	A	B	C	C
4	A	A	C	C	C	C	C	C	C	C	C	C	C	C	C
5	A	B	B	C	C	C	B	B	C	C	C	B	B	C	C
6	A	B	B	B	C	A	A	B	B	C	A	A	B	B	C
7	A	A	B	C	C	A	A	B	C	C	A	A	B	C	C
8	A	A	C	C	C	C	C	C	C	C	C	C	C	C	C
9	A	B	B	C	C	C	B	B	C	C	C	B	B	C	C
10	A	B	B	B	C	A	B	B	B	C	A	B	B	B	C
11	A	A	B	C	C	A	A	B	C	C	A	A	B	C	C
12	A	C	C	C	C	C	C	C	C	C	C	C	C	C	C

(4) Allocation with 7th priority list



Other Test Results

- 10-second CPU time limit vs. Unlimited
 - Same Answer in 10 out of 12 Cases
 - Other Cases

Extra Revenue w/o time limit

Case 1: 28 = 1.04%

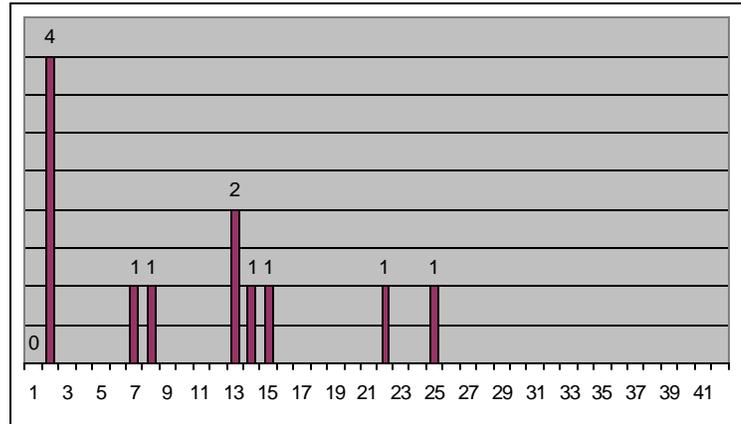
Case 2: 176 = 6.20%

Extra Value w/o time limit

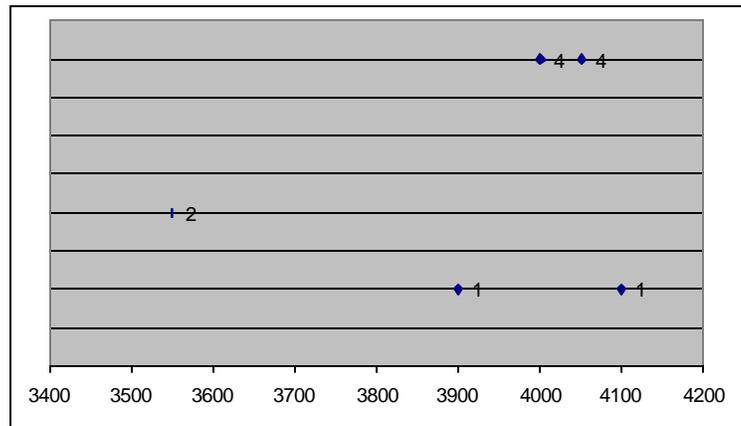
Case 1: -36 = -0.90%

Case 2: 10 = 0.25%

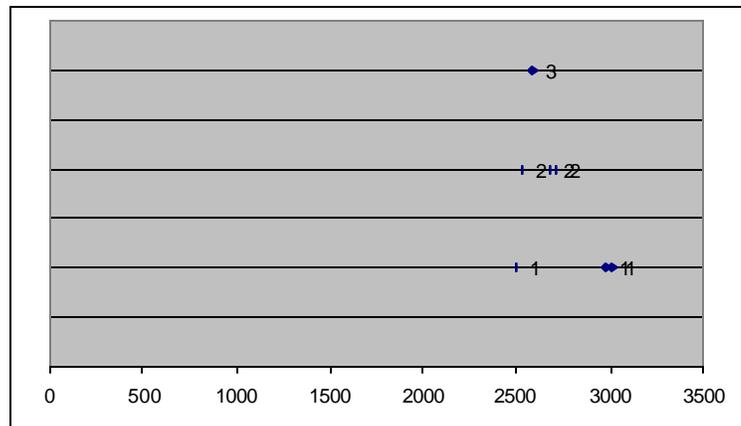
- Priority Level when the Final Result is Reached



- Total Value Achieved (with unlimited time)



- Revenue (with unlimited time)



Conclusions

- It Works.
 - All prioritized combinations are considered in less than 26.7 CPU seconds.
 - On 40 MHz Sun Sparc 1000 machine with CPLEX 6.0 (not 6.5).
- Bidders Can Do It.
- Testing Continues...