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Issues in FCC Package Bidding
Auction Design

FCC Wye River Conference III
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Joint work with
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Decisive Analytics Corporation
(under contract with Computech, Inc.)
November 22, 2003
Outline of Talk

• Examine ISAS auction design
  – No provision for “last and best”
  – Is chosen linear pricing algorithm most appropriate?
  – Communication complexity

• Consider using ascending proxy as final “round”
  – Address computational issues
  – Design of accelerated proxy mechanism

• Test alternative linear pricing approaches
  – Used accelerated proxy mechanism to benchmark linear pricing algorithms

• Bidder aid tools
Positives of the ISAS Auction Design

- Price discovery
- Package creation
- No budget exposure problem (XOR)
- Linear pricing
  - Perceived as fair
  - Easy to use
  - Reduces parking problem
- Transparency
Open Issues with the ISAS Auction Design

- May require large increment size to close in reasonable time
- No provision for “last and best”
- Limited testing of linear pricing scheme
- Bidders must determine what packages to create and bid
- Rules may seem complex to bidder
- Treats every item as unique
  - Better to have quantity specification for homogeneous items
- Opportunity for gaming
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  - Design of *accelerated* proxy mechanism

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Economic Characteristics of Ascending Proxy

• Guaranteed to arrive at efficient outcome
• When buyer sub-modularity property holds, mechanism arrives at VCG prices
• Even when buyer sub-modularity property does not hold, prices are in the core
• Collusion and other destructive bidding eliminated since bidders forced (through proxy) to bid straightforwardly
Ascending Proxy Mechanism

• Each bidder provides all packages of interest to proxy with valuations
• Bidder can only win one of the packages submitted (XOR among packages of bidder)
• Proxy bids for bidder in myopic best-response manner
• Auctioneer solves WDP to determine provisionally-winning bids
• If bid is non-winning, then price goes up by epsilon
• Proxy agents place bids until no bids are profitable or winning
• Auction ends when no new bids are placed in a round
• At end of auction, winning bidders pay what they bid
Proxies Place Bids

- A bidder’s proxy follows a “Myopic Best Response” strategy
  - Myopic because the proxy only looks at the current prices
  - Best response refers to profit maximizing
    - $Profit = Value - Price$

- In a round, a proxy submits the bidder’s most profitable package at the current price
  - If ties exist, all ties are submitted
  - If a bidder has a current provisionally winning bid the proxy does not place any new bids (since all non-winning bids of that bidder are not as profitable as the winning bid)
Proxy Rounds

- Simulation of a Proxy Auction with 6 licenses and 10 bidders
  - Most bidders entered many packages ~ 30-40 packages (out of possible 63)
  - Value of the auction ~ $3.4 million
- Results:
  - With $5000 increment, over 22,000 rounds
  - With $10 increment, over 9 million rounds!
- Auction theory requires very small increment
- But, FCC needs an auction design that can handle thousands of items

*Is there a way to overcome this computational stumbling block?*
**Accelerated Proxy Mechanism**

- Reduces substantially the number of rounds of the proxy mechanism
- Works backwards from “end result” and thereby requires far fewer iterations than proxy mechanism
- Same nice properties as Ausubel-Milgrom proxy auction
Accelerated Proxy: Methodology

STEP 1: Solve Winner Determination Problem for Efficient Outcome
(Objective function coefficients are valuations)
• Determines winning bidders
• Determines winning bids of winning bidders

STEP 2: Determine the Opening Prices for All Bids of All Bidders
  a. Opening prices of non-winning bidders’ bids = valuations
  b. Opening prices of winning bids of winning bidders = “Safe Price”
     Safe Price = Max of all valuations on this package by non-winning bidders
     Opening Price (Winning Bid) = Safe Price
  c. All opening prices of all losing bids of winning bidder have same profitability
     Profit (Winning Bid) = Valuation (Winning Bid) - Opening Price (Winning Bid)
     Opening Price (Non-Winning Bid) = Valuation (Non-Winning Bid) - Profit (Winning Bid)

STEP 3: Use Increment Scaling Method to Determine Optimum Prices
**Accelerated Proxy: Increment Scaling**

**FIRST STAGE:** Set increment size to some large increment (scale all opening prices down to the nearest increment, but not less than zero)
- Implement Proxy Mechanism until auction ends with no new bids

**EVERY SUBSEQUENT STAGE:**
- Given final outcome from prior stage, check if the current increment satisfies the "increment threshold"
  - If threshold met **STOP**, **ELSE**:
- Determine starting point for the next stage
  - Every winning agent’s price vector is set equal to their final bid amounts from the previous stage less the amount of the current increment. Every non-winning agent’s price vector is set equal to their prior bid amounts
- Scale down the current increment by a factor of 10 and start the next stage
- **NOTE:** May need “Corrective Rollback”
Properties of Accelerated Proxy

- Efficient Outcome
- Buyer Pareto-optimal payments by winners when the “agents-are-substitutes” property holds
- Buyer Pareto-optimal payments even when the “buyer sub-modularity” property does not hold
- Forces straight-forward bidding and therefore removes opportunity for shill bidding and collusion
- Requires far fewer integer optimizations than a direct application of the ascending proxy auction
  - Bounded by a function of number of digits of accuracy required, number of packages in the optimal allocation and number of bids by winning bidders
- Obtains core outcome when agents-are-substitutes property does not hold
Rounds: Proxy vs. Accelerated Proxy

- Accelerated proxy achieves efficient outcomes with bidder payments accurate to 1 cent
- Proxy accurate to within $5,000
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• Bidder aid tools
Testing Linear Pricing against Proxy

• Created a number of small test cases and 10 larger profiles
  – 6 items, 10 bidders, approx. $3M revenue

• Tested:
  – Ausubel-Milgrom Ascending Proxy
  – Accelerated Proxy
  – Three Linear Pricing Algorithms (with myopic best response bidding and fixed increments)

• Compare:
  – Outcomes (efficiency)
  – Payments
  – Speed of auction
Pricing Algorithms

- **RAD** (DeMartini, Kwasnica, Ledyard and Porter)
- Smoothed Anchoring (FCC)
- Smoothed Nucleolus
  - RAD first stage
  - Smoothing second stage
Test Case 1: Agents Are Substitutes

<table>
<thead>
<tr>
<th>Agent</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Package</td>
<td>AB</td>
<td>BC</td>
<td>C</td>
<td>C*</td>
<td>AB*</td>
</tr>
<tr>
<td>Value</td>
<td>21</td>
<td>35</td>
<td>14</td>
<td>20</td>
<td>22</td>
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<table>
<thead>
<tr>
<th>Method</th>
<th>Increment</th>
<th>Rounds</th>
<th>Revenue</th>
<th>A4, {C}</th>
<th>A5, {AB}</th>
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<tr>
<td>Accelerated Proxy</td>
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<td>35</td>
<td>14</td>
<td>21</td>
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<tr>
<td>Proxy</td>
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<td>298</td>
<td>35.05</td>
<td>13.99</td>
<td>21.06</td>
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<tr>
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<td>298</td>
<td>35.05</td>
<td>13.99</td>
<td>21.06</td>
</tr>
<tr>
<td>RAD</td>
<td>0.1</td>
<td>291</td>
<td>35.02</td>
<td>14.03</td>
<td>20.99</td>
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<tr>
<td>VCG</td>
<td>-</td>
<td>-</td>
<td>35</td>
<td>14</td>
<td>21</td>
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</table>

*Buyer sub-modularity fails*
Test Case 2: Agents Are Not Substitutes

<table>
<thead>
<tr>
<th>Agent</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Package</td>
<td>AB</td>
<td>BC*</td>
<td>AC</td>
<td>A*</td>
</tr>
<tr>
<td>Value</td>
<td>20</td>
<td>26</td>
<td>24</td>
<td>16</td>
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<table>
<thead>
<tr>
<th>Method</th>
<th>Increment</th>
<th>Rounds</th>
<th>Revenue</th>
<th>Payments by winning agents</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A2, {BC}</td>
<td>A4, {A}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Accelerated Proxy</td>
<td>0.01</td>
<td>16</td>
<td>24</td>
<td>17</td>
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<tr>
<td>Proxy</td>
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<td>24.2</td>
<td>12.1</td>
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<tr>
<td>Smoothed Anchoring</td>
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<td>234</td>
<td>24.33</td>
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<tr>
<td>Smoothed Nucleolus</td>
<td>0.1</td>
<td>234</td>
<td>24.33</td>
<td>12.19</td>
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<tr>
<td>RAD</td>
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<td>257</td>
<td>23.95</td>
<td>8.3</td>
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<tr>
<td>VCG</td>
<td>-</td>
<td>-</td>
<td>8</td>
<td>8</td>
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</table>
## Summary of 10 profiles

**$5000 increment, 6 items, 10 bidders, $3M auction**

<table>
<thead>
<tr>
<th>Profile</th>
<th>Number of Winning Packages</th>
<th>Agents are Substitutes?</th>
<th>Efficient Result?</th>
<th>Revenue within tolerance ($5,000)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Proxy</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>YES</td>
<td>All methods</td>
<td>YES</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>NO</td>
<td>RAD only</td>
<td>YES</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>YES</td>
<td>All methods</td>
<td>YES</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>NO</td>
<td>None</td>
<td>YES</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>NO</td>
<td>All but Proxy</td>
<td>YES</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>NO</td>
<td>All but RAD</td>
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</tr>
<tr>
<td>7</td>
<td>2</td>
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<td>All methods</td>
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<td>8</td>
<td>3</td>
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<td>All methods</td>
<td>$13,000</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>YES</td>
<td>RAD only</td>
<td>$8,000</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>YES</td>
<td>None</td>
<td>YES</td>
</tr>
</tbody>
</table>
• Accelerated proxy achieves efficient outcomes with bidder payments accurate to 1 cent
• Linear pricing schemes use an increment of $5,000
## Average Performance of the Pricing Schemes

<table>
<thead>
<tr>
<th>Method</th>
<th>Average Number of Rounds (Increment Size: $5000)</th>
<th>Abs. Revenue Deviation from Accelerated Proxy Revenue ($)</th>
<th>Abs. Price Deviation from Accelerated Proxy Price ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Median</td>
<td>Max.</td>
</tr>
<tr>
<td>Proxy</td>
<td>21,260</td>
<td>4,551</td>
<td>3,683</td>
</tr>
<tr>
<td>Smoothed Anchoring</td>
<td>526</td>
<td>4,828</td>
<td>2,483</td>
</tr>
<tr>
<td>Smoothed Nucleolus</td>
<td>527</td>
<td>4,539</td>
<td>2,161</td>
</tr>
<tr>
<td>RAD</td>
<td>562</td>
<td>5,446</td>
<td>2,508</td>
</tr>
</tbody>
</table>

**Accelerated Proxy:** 537 rounds on average for accuracy to **1 cent**
Conclusions of Testing

• Linear pricing arrives at outcomes similar to that of ascending proxy when increment the same, except when synergies are very large
• No linear pricing algorithm dominates all others
• With linear pricing, need some type of smoothing to overcome fluctuations
• *Accelerated* ascending proxy much faster than any other approach for same accuracy
Pros and Cons of Accelerated Proxy

• Pros:
  – Efficient
  – Core Outcome
  – No Gaming
  – Limits bidder participation burden
  – Computationally competitive for greater accuracy
  – Verifiability possible without disclosing valuations

• Cons:
  – Bidders must provide valuations
  – Language (Puts burden on bidder)
    • SOLUTION: Bidder aid tools
  – No Feedback (Price discovery)
    • SOLUTION: Hybrid designs
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A Need for Bidder-Aid Tools

- How does the bidder express his business plans in a compact way?
- How does one create packages that reflect business needs?
- How does one alter business plans based on price discovery?
Bidder-Aid Tool Concept

Bidder → Tool

- Language X
- Items, relationships, values

Tool → Auction System

- XOR Language
- Bidder's "best" bids
Example of Bidding Language: Cramton

- **Items in a given class are in terms of $/MHz-pop**
  - May want more than one class: (e.g. Large cities, small cities, rural areas)

- **Equivalence classes**
  - A minimum amount of MHz needed
  - A value (above norm) for certain bands
  - A bonus for blocks that are contiguous
  - Incremental values for each increment above the minimum required

- **Minimum and maximum amounts of total population needed**

- **Budget constraints (Possibly more than one)**

- **Secondary items:**
  - Contingent items (only want A if coupled with B)
  - Synergy (Want A with stand-alone value; but if with B, A gets synergy value)

The Language is translated into an optimization problem that determines the “best” packages for this bidder given budget, current prices, and activity rules
Generating Proposals: Example of Optimization

Maximize \( \sum_{b \in B} (v_b - p_b)x_b \)

Subject to:

\( \sum_{j \in B^l} x_j \leq 1 \) for all items \( l \)

\( \sum_{i \in B^c} x_i \leq |B^c| x_c \) for all contingent sets "c"

\( \sum_{c=1}^{\# \text{Con. Sets}} x_c \leq 1 \)

\( \sum_{b \in B} \text{BidAmt}_b x_b \leq \text{Budget} \)

\( x_b \in \{0,1\} \) for all bids
Conclusions

• Linear pricing with smoothing works well
• Further work on bidder aid tools is needed
• Other issues with ISAS design
  – Opportunity for gaming (signaling)
  – XOR bidding language forces explosion of bids for homogeneous items
  – Lots of bidder participation during auction
• Can other hybrid designs overcome these issues?
  – Clock Auction followed by Proxy
  – Iterative Proxy
• Issues with hybrid designs:
  – Activity rules
  – Information to bidders
  – What information passes between stages
Package Bidding: Bidders’ Needs

- Easy to understand rules
- Easy to express needs
- Easy to interpret results
- Fair
- Reasonable completion time
- Price discovery
- Risk/Exposure not excessive
- Ability to compete effectively
Package Bidding:  FCC Perspective

- **Efficiency** – Spectrum will be used
- **Transparency** – No security issues
- **Fairness** – Spectrum not held hostage to law suits
- **Speed** – Spectrum is allocated quickly
- **Participation/Competition** – Buyers come to auction
QUESTIONS?
Properties: AAS and BSM

Agents-Are-Substitutes (AAS) if:

\[ w(A) - w(A \setminus K) \geq \sum_{a \in K} [w(A) - w(A \setminus a)], \quad \forall K \subset A, \ 0 \notin K \]

- VCG payoffs are supported in the core only when AAS condition is satisfied

Buyer Sub-Modularity (BSM) if:

\[ w(L) - w(L \setminus K) \geq \sum_{a \in K} [w(L) - w(L \setminus a)], \quad \forall K \subset L, \ 0 \notin K, \]

for all \( L \subseteq A, \ 0 \in L \)

- For all sub-coalitions, the incremental value of an additional member is decreasing in the coalition size
- BSM is a stronger condition