Winner Determination in Combinatorial Exchanges

Tuomas Sandholm
Associate Professor
Computer Science Department
Carnegie Mellon University
and
Founder, Chairman, and Chief Technology Officer
CombineNet, Inc.
Outline

• CombineNet company overview
• Performance on real-world combinatorial procurement auctions
• Exchange formulation & problem hardness
• Exchange instance generator
• Experiments with different solution technologies & instance types
• Factors affecting problem difficulty
• Discussion of the expected FCC exchange model
CombineNet, Inc.

- Leading vendor of markets with expressive competition
- Technology development started 1997
- Company founded April 2000
- 55 full-time employees and 9 professors
  - Tuomas Sandholm, Subhash Suri, Egon Balas, Craig Boutilier, John Coyle, Holger Hoos, George Nemhauser, David Parkes, Rakesh Vohra
- 1 patent issued and 13 pending
  - Bidding languages
  - Market designs
  - Algorithms
  - Preference elicitation
  - Methods around basic combinatorial bidding that make it practical
  - ...
- Headquartered in Pittsburgh, with offices in London, San Francisco, Atlanta, Brussels
CombineNet event summary (latest 2 years)

- ~100 combinatorial procurement auctions fielded
  - Transportation: truckload, less than truckload, ocean freight, air freight
  - Direct sourcing: materials, packaging, production
  - Indirect sourcing: facilities, maintenance and repair operations, utilities
  - Services: temporary labor
  - …

- **Total transaction volume: $6 B**
  - Individual auctions range from $8 M to $730 M

- **Total savings: $1.02 B**
CombineNet applied technologies

• Operations research
  – LP relaxation techniques
  – Branch and bound, Branch and cut
  – Multiple (efficient) formulations
  – ...

• Artificial intelligence
  – Search techniques
  – Constraint propagation
  – ...

• Software engineering
  – Modularity supports application of most appropriate solving techniques and refinements, some of which depend on problem instance
  – C++ is effective (fast) implementation language, STL is indispensable
  – XML is effective (extensible) input/output metalanguage
    • Off-the-shelf XML parsers are too slow and heavy for large (100s of MB) inputs, so we built our own
  – ...
The largest expressive competition problem we have encountered is the transportation services procurement auction, involving:

- Approximately 3000 trucking lanes to be bought, multiple units of each.
- Around 120,000 bids, no package bids.
- Approximately 130,000 side constraints.
- CPLEX did not solve in 48 hours.
- Our technology clears this optimally & proves optimality in 4½ minutes.
One of the **hardest** expressive competition problems we have encountered

- Transportation services procurement auction
- 22,665 trucking lanes to be bought, multiple units of each
- 323,015 bids, no package bids
- 8 max winners constraints (overall & regional)
Combinatorial exchanges
Combinatorial exchanges are a key effort at CombineNet

- CombineNet has ~40 engineers, almost half of whom work on winner determination technology
- The main backend hosted product, ClearBox, does combinatorial auctions, reverse auctions, and exchanges
  - With hundreds of types of side constraints
  - With multiple attributes and a fully expressive language for taking them into account
- $1.84 M NIST ATP grant for a 3-year effort for speeding up combinatorial exchanges
  - One year completed
- Fastest engine (by 1-2 orders of magnitude) for clearing combinatorial exchanges
Exchange model formulation
(simple formulation without side constraints shown)

\[
\max \sum_{j \in B} p_j x_j = \text{surplus (alternatively, could maximize liquidity)}
\]

such that

\[
\sum_{j \in B} q_{ij} x_j \leq 0 \quad \forall i \in I
\]

where

- \(I\) is the set of items \(i\)
- \(B\) is the set of bids \(j\)
- \(x_j\) is the (binary) decision variable for bid \(j\)
- \(p_j\) is the price of bid \(j\)
- \(q_{ij}\) is the quantity of item \(i\) in bid \(j\)

quantities are positive for demand, negative for supply

Sandholm ICE-98, AAAI-99 workshop on AI in Ecommerce, AGENTS-00, CI-02
Sandholm & Suri AAAI-00, AIJ-03
Exchange problem hardness
[Sandholm, Suri, Gilpin & Levine AAMAS-02]

- Thrm. NP-complete
- Thrm. Inapproximable to a ratio better than $\#bids^{1-\varepsilon}$
- Thrm. Without free disposal, even finding a feasible (non-zero trade) solution is NP-complete
Exchange instance generator

- Model of item co-occurrence: building a bundle for a bid

- Each bidder has his own subgraph of items
- Each item in a bidder’s subgraph is only bought or sold by that bidder
- Complementarity in bids and substitutability in asks determined by edges between items in bundle
  - Edges assigned weights, sum of weights on a node’s edges provides factor used in calculation
Example of pricing bundle bids in the instance generator

- **Items in the bundle**
  - Bidder action: Buy, Buy, Sell, Sell
  - Item quantity ($\alpha = 0.6$): 3, 1, 4, 1
  - Market Price: 2.34, 9.01, 6.53, 0.14
  - Bidder’s Price (+/- 25%): -5%, +7%, +21%, -16%
  - Bid Price (+/- 3%): -1%, +1.5%, +2.5%, -1.5%
  - Graph factor: +2%, +2%, -3%, -1%
  - Final Price = -14.55 = 3 * 2.25 + 1 * 9.98 - 4 * 7.79 - 1 * 0.12

- **Ask bid at $14.55**
Exchange experiment setup

- **Basics about instances**
  - 50 items, 10 bidders, 50 bids per bidder (= 500 bids)
  - Each bid must be accepted all or nothing
  - Bundle bids permitted, with average of 2.5 items per bundle
  - Multi-unit, with average item quantity of 2.5
  - Free disposal permitted by buyers and sellers
  - Exchange types: 1) Buyer/Seller, 2) Pure bids, 3) Buy&Sell
  - All runs completed in under 3 hours

- **Constraints**
  - Max winners constraint for whole exchange
    - At most 5 of 10 bidders accepted
  - Cost constraint for one bidder
    - First bidder is awarded at least 20% of market by $ value
  - Discount schedule for one bidder
    - Percentage discounts based on $ awarded
Speed of different solution technologies

• All timing results are for finding an optimal allocation & proving optimality
• Solution technologies compared
  – CPLEX 8.1 out-of-the-box vs. CombineNet’s technology
    • Tuned CPLEX is within 10% of CPLEX out-of-the-box
• Results over all exchange types

<table>
<thead>
<tr>
<th>Solution Technology</th>
<th>Avg run time (60 instances)</th>
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<tbody>
<tr>
<td>CPLEX</td>
<td>400 s</td>
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<tr>
<td>CombineNet technology</td>
<td>27  s</td>
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Speed by instance type

- All exchanges, constrained vs unconstrained
  
<table>
<thead>
<tr>
<th></th>
<th>CONSTRAINED</th>
<th>UNCONSTRAINED</th>
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<tr>
<td>CPLEX</td>
<td>408 s</td>
<td>393 s</td>
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<tr>
<td>CombineNet technology</td>
<td>29 s</td>
<td>24 s</td>
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- All exchanges, different exchange types
  
<table>
<thead>
<tr>
<th></th>
<th>BUYER/SELLER</th>
<th>PURE</th>
<th>BUY&amp;SELL</th>
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<td>CPLEX</td>
<td>349 s</td>
<td>164 s</td>
<td>689 s</td>
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<tr>
<td>CombineNet technology</td>
<td>19 s</td>
<td>14 s</td>
<td>47 s</td>
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Factors that affect problem difficulty

In order of impact:

• **Amount of demand for a given item**
  – Higher average bid item quantities make problems much harder
  – Single-unit exchanges are much less complex than multi-unit exchanges

• **Competitiveness of bids**
  – Close bid prices make problem much tougher
    • More possible solutions are close in value

• **Side constraints**
  – May either help or hurt, depending on the problem and constraints
  – Usually hurt, but not relatively as much as in reverse auctions

• **Free disposal**

• **Size of subset of items bidder is interested in**
  – Larger subsets will mean there are more bidders on each item
  – The more bidders on an item, the tougher the problem

• **Buy&Sell bundles**
Conclusions

• Combinatorial markets of different types have become a reality and CombineNet has a lot of experience designing, building, fielding & hosting them

• Combinatorial exchanges are very complex to clear
  – NP-complete, inapproximable
  – Orders of magnitude more complex than combinatorial auctions or reverse auctions of the same size

• CombineNet technology is the fastest for the problem by 1-2 orders of magnitude

• Optimal clearing scales to reasonable problem sizes

• Complexity depends on certain features of the instances, as presented
Expected FCC exchange model

• **General points**
  – Each license for a frequency range in a region is an item
  – There are # ranges (~35) X # regions (500?) items

• **Aspects that decrease complexity**
  – Each item has a single unit only
  – There is a single seller for each item (though multiple buyers possible)
  – There is a definite structure to bids, by region and frequency range
  – Small sellers and large buyers provide asymmetry

• **Aspects that increase complexity**
  – Substitutability of frequency ranges may explode the size of bids
  – Large bundles are likely for the buyers
  – Potentially several large buyers for each item