Towards Iterative Combinatorial Exchanges

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Motivation

- Highly fragmented spectrum (frequency, control, and geography) ⇒ proposal for “big-bang” exchange

- Assumptions:
  - forced relocation of spectrum to alternative bands still leaves a substantial bargaining problem, and high transaction costs
  - efficient reallocation is the main goal (although “reasonable” FCC revenue important)
Combinatorial Exchanges

- **Multiple buyers and sellers, w/ expressive bids**
  - e.g. “Buy 10MHz in NYC counties A, B, C and D for $1 million”, “Sell 78-84 MHz in counties A and D for $300,000”

- FCC can also participate, **actively**:
  - e.g. the *only* agent able to buy ITFS licenses and convert into flexible-use licenses

  and **passively** (define aggregations):
  - e.g. “all contiguous 6 MHz blocks of spectrum in a BTA are equivalent”
Main Challenges

- Winner-determination
  - likely to be harder than one-sided auctions (Sandholm’s talk)

- Economic
  - mitigating the bargaining or “hold-out” problem

- Preference elicitation
  - hard valuation problems
  - iterative designs likely important to guide elicitation
Main Challenges

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11/23/2003 Combinatorial Exchanges
Bargaining Problem.

Example:

Many \textit{ex post} Nash equilibrium:
- \((5,15,20)\); \((10,10,20)\); \((15,15,30)\)...
- presents an \textit{efficiency problem}, because agents need to select an equilibrium.

\textbf{Construct \textit{ex post} Nash:}
- allocate \(\pi_i\) to some agent \(i\), with \(V(N)-V(N\setminus i)>0\)
- adjust \textit{values}, and repeat.
A One-Shot Design

(Parkes, Kalagnanam and Eso, 2001)

- Collect bids
- Compute $V(N)$, value of surplus-maximizing trade given all bids
  - implement this outcome
- Compute $V(N\setminus i)$, value of surplus-maximizing trade without bids from $i$
- Divide surplus $\sum_i \pi_i = V(N)$ across participants
  - try to mitigate bargaining problem
Surplus Division

Allocate payoffs, $\pi_i \geq 0$, to satisfy:

\[
\sum_i \pi_i \leq V(N) \quad \text{(BB)} \\
\pi_i \leq V(N) - V(N \setminus i), \quad \forall \ i \quad \text{(*)} \\
\pi_i \geq 0, \quad \forall \ i \quad \text{(P)}
\]

(*) is just ($\sum_i \pi_i = V(N)$ and $\sum_{j \neq i} \pi_j \geq V(N \setminus i)$)

**Lemma.** Any mechanism that implements $V(N)$ and satisfies (BB), (*), and (P) has *ex post regret* $\pi_{VCG,i}$ for agent $i$, given bids of other agents.
Consider VCG-based schemes. Set objective to \( \min D(\pi, \pi_{VCG}) \), for distance \( D(.,.) \).

- **Threshold.** Minimize worst-case \((\pi_{VCG,i} - \pi_i)\)
  - i.e. minimize the maximal ex post regret.

- **Fractional.** Each agent gets \( \pi_i = \mu \pi_{VCG,i} \)

- **Large.** Allocate payoff in order \( \pi_1, \pi_2, \pi_3 \ldots \)

- **Reverse.**
Stylized Representations

- **vickrey**
- **regular**
- **frac**
  - grad $1 - \mu$
- **bid price**
- **reverse**
- **large**
- **small**
- **thresh**
Threshold: Special Cases

- Implements the k-DA uniform price, double auction with $k=0.5$ (Wilson’85)
  - Threshold payoff division implemented with price $p^*=0.5\left(\min(a_{k+1},b_k)+\max(b_{k+1},a_k)\right)$, asks $a_1<a_2<\ldots<a_m$, bids $b_1>b_2>\ldots>b_m$, $k$ items trade

- Second-best (for efficiency) for the standard single item bargaining problem, for iid and Uniform $[0,1]$ values and costs (Myerson & Satterthwaite, 83)
Experimental Validation

- **Limited strategy space:**
  - \( b_i(S) = (1 - \alpha) v_i(S), \forall S, \) if buyer
  - \( b_i(S) = (1 + \alpha) v_i(S), \forall S, \) if seller

- **Compute a symmetric ex ante BNE:**
  \[
  \alpha^* = \arg \max_{\alpha} E_i E_{-i} \left[ v_i(x(\alpha, \alpha^*)) - p_i(\alpha, \alpha^*) \right]
  \]

  \( x^*(\alpha, \alpha^*) \) is the allocation,
  \( p_i(\alpha, \alpha^*) \) is payment to agent \( i \).
**Naive Approach**

- Enumerate a payoff matrix, compute *ex ante* BNE

\[
\begin{pmatrix}
-0.5 & -0.48 & \cdots & 0 & \cdots & 0.12 & 0.14 & \cdots & 0.98 & 1.0 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
-0.5 \\
-0.48 \\
\cdots \\
0 \\
\cdots \\
0.12 \\
0.14 \\
\cdots \\
0.98 \\
1.0
\end{pmatrix}
\]

Took **2.5 days**, for a grid size of **0.01**, 500 instances, 5 buyers, 5 sellers, 20 goods, 10 bids/asks per agent.
Iterative Approach.

α = 0.18

α = 0.22

α = 0.26

Combinatorial Exchanges
Algorithm

(w/ David Kyrch)

Choose a small set of strategies
\( A^t = (\alpha_1^t, \ldots, \alpha_M^t) \).

Assume all agents except agent 1 play
\( \alpha^t \in A^t \)

Compute the BR, \( \alpha^* \in A^t \), given \( \alpha_t \)

Move \( \alpha^t+1 \) towards \( \alpha^* \)

Refine \( A^t \) to focus search.
Details.

- Select 7 points in A
- New center: \( \alpha^{t+1} = \frac{1}{3} \alpha^t + \frac{2}{3} \alpha^* \)
- Select a new range \( A^{t+1} \), centered on \( \alpha^{t+1} \)
  - \( |A^{t+1}| = \frac{3}{4} |A^t| \), if \( \alpha^{t+1} \) within current range
  - \( |A^{t+1}| = 4(\alpha^{t+1} - \alpha^t) \), otherwise.

- Terminate when \( \alpha^* \) is within 0.01 of \( \alpha^t \)
  - finally validate that \( \alpha^* \) is a BR to \( \alpha^* \) over entire range [-0.5,1.0]
Experimental Results

- 5 buyers, 5 sellers, 20 goods
- 10 bundles/agent.
  - Uniform (Sandholm’99), XOR valuations.
- 500 instances

- Compute 1% accuracy in 2.5 CPU hours.
Example 1- VCG payments
Example 2- No Discount
Example 3- Large
Example: Validity

Validating *ex post* Nash of Threshold rule
Main Results

Equilibrium Efficiency

Vickrey
Large
Threshold
Fraction
Reverse
Small
Equal
No Discount
Optimal strategy in **Large** is to overbid
- at least one participant has negative ex post payoff in BNE
- an agent in efficient allocation can bid $v + \Delta$, large $\Delta$, and ensure $\pi_{VCG,i}$

Buyers in **Threshold** can only benefit by decreasing their bid, and then only if
- their bid is adjusted by more than their Threshold payoff, or
- there is some $V(N \setminus j)$, $j \neq i$, without $i$. 

(99% efficient) (95% efficient)
FCC: A Special Player

(Milgrom)

- Can also apply core constraints for the FCC
  \[ \pi_{FCC} + \sum_{i \in L} \pi_i \geq V(FCC \cup L), \quad \forall \ L \subseteq (N \setminus FCC) \]

- FCC cannot propose an alternative with more revenue that a subset of participants will all prefer (based on their reports).

- Helps to prevent “give aways.”
Main Challenges

- Winner-determination
  - likely to be harder than one-sided auctions
- Economic
  - mitigating the bargaining or “hold-out” problem
- Preference elicitation
  - hard valuation problems
  - iterative designs likely important to guide elicitation
Elicitation for Exchanges: Key Problems.

- Item discovery
  - scope of exchange may not be initially known

- Price discovery
  - may be *no trade* in initial stages

- Bargaining
  - the bargaining problem is omnipresent
Threshold Information Requirements.

Consider information:
\[ v'_i(S) = \max(0, v_i(S) - \Delta_i) \]

Can compute Threshold with:

1. Complete info from all losers
2. Winner i in \( V(N \setminus j) \) for all \( j \neq i \), or bids \( \Delta_i = 0 \)
3. Winner i receives \( \pi_i > 0 \) from Threshold, or bids \( \Delta_i = 0 \)
Staged Approach.

Proxy: \( \overline{v}_{\text{bid}}(S) \) \( \overline{v}_{\text{ask}}(S) \)

Threshold

User

activity rules

\[ \text{Outcome}(\overline{v}_{\text{bid}}(S), \overline{v}_{\text{ask}}(S)) \]

prices \( \overline{p}_{\text{bid},j}, \overline{p}_{\text{ask},j} \)

next stage

\[ \text{Outcome}(v_{\text{bid}}(S), v_{\text{ask}}(S)) \]

prices \( p_{\text{bid},j}, p_{\text{ask},j} \)
High-Level Approach.

- **Proxied:**
  - *direct* but *incremental* value information.

- **Threshold:**
  - implement the Threshold rule in each stage

- **Activity Rules:**
  - consistent bounds across stages (relax by $\alpha$?)
  - require *progress* across stages

- **Staged w/ Final Round.**
  - price-based feedback
Proxy Information

- Lower and upper valuation functions provided w/ appropriate bidding language
  - Maintain consistency, w/ \( \overline{v}(S') \geq \overline{v}(S), \forall \ S' \supseteq S; \ \underline{v}(S') \leq \underline{v}(S), \forall \ S' \subseteq S \)

- Incremental tightening of information allows early price discovery
Activity Rules.

- **Consistency.**
  - Can refine bounds on existing bundles
  - Can introduce new bundles (w/ bounds to respect free disposal)

- **Progress:**
  - Tighten limits on allowed slack between bounds in later stages
  - Limit # of additional bundles that can introduced in later stages

- At some point, move to a final stage.

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Combinatorial Exchanges
In Each Stage...

- Compute “high” Threshold outcome w/ high bids and low asks
  - provides feedback in early stages

- Compute “low” Threshold outcome w/ low bids and high asks
  - provides feedback in later stages

- Finally implement this outcome
Buy-side: High item prices

- Compute high bid prices $\overline{p}_{\text{bid},j}$ for items $j$ based on high bids $\overline{v}_i(S)$
- Provide accurate winner feedback, suggest how far can drop price and still win.

$$\min_{p, \delta} \delta$$

s.t. $\overline{v}_i(S') \geq \sum_{j \in S'} \overline{p}_{\text{bid},j}$  \forall \text{ winner } i, \text{ winner } S'$

$\overline{v}_i(S) \leq [\overline{v}_i(S') - \sum_{j \in S'} \overline{p}_{\text{bid},j}] + \delta + \sum_{j \in S} \overline{p}_{\text{bid},j}$  \forall \text{ winner } i, \text{ loser } S$

$\overline{v}_i(S) \leq \delta + \sum_{j \in S} \overline{p}_{\text{bid},j}$,  \forall \text{ loser } i

(assumes an XOR bidding language, might also want to do smoothing across stages.)
Buy-side: Low item prices

- Compute low bid prices $p_{\text{bid},j}$ for items $j$ based on low bids $v_i(S)$.
- Provide accurate loser feedback, suggest how far must increase price to win.

$$\min_{p,\delta} \delta$$

s.t. $v_i(S) \leq \sum_{j \in S} p_{\text{bid},j}$ \quad \forall \text{loser } i$

$v_i(S') \geq \sum_{j \in S'} p_{\text{bid},j} - \delta$ \quad \forall \text{winner } i, \text{winner } S'$

$v_i(S) \leq [v_i(S') - \sum_{j \in S'} p_{\text{bid},j}] + \delta + \sum_{j \in S} p_{\text{bid},j}$ \quad \forall \text{winner } i, \text{loser } S$
Sell-side: Item prices

- Compute **low ask prices** $p_{ask,j}$ to give winner feedback, suggest how far can increase price and still win
  - make these prices accurate for winners, with $v_i(S') \leq \sum_{j \in S'} p_{ask,j}, \forall \text{ winners } (i, S')$

- Compute **high ask prices** $\bar{p}_{ask,j}$ to give loser feedback, suggest how must drop price to win
  - make these prices accurate for losers, with $\bar{v}_i(S) \geq \sum_{j \in S} \bar{p}_{ask,j}, \forall \text{ losers } i$
Item Discovery.

- Also need buy-side prices for items offered on sell-side
  - perhaps $0.5(p_{ask,j} + \overline{p}_{ask,j})$ is a good signal?

- Also need sell-side prices for items requested on buy-side
  - perhaps $0.5(p_{buy,j} + \overline{p}_{buy,j})$ is a good signal?
Next steps (practical).

1. Put together a computer-based simulation of this system.
2. Implement simple bidding agents, check for bad behaviors, refine.
3. Implement more sophisticated bidding agents, check for bad behaviors, refine.
   - Work on computational properties, provide scalability.
4. Run in an Experimental Economics Lab?
Conclusions.

- A **combinatorial exchange** can facilitate a “big bang” spectrum auction; allow incumbents and new entrants to trade

- **Key issues** are:
  - computational
  - economic (bargaining problem)
  - preference elicitation

- Proposed a straw-model design, lots of interesting questions going forward!