An Experimental Comparison of the Simultaneous Multi-Round Auction and the CRA Combinatorial Auction

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Executive Summary

We use laboratory environments identical to those discussed in our SMR report (*An Experimental Analysis of the Federal Communications Commission's Eligibility Rules, Nov. 10, 1999*) to test one version of the CRA Combinatorial (Combo) auction. We conducted 49 auctions in which 6 to 8 subjects bid for 10 available licenses. The payoffs to subjects in these environments were generated by varying the synergy value among licenses (additive, low, medium and high) and the package density (average number of licenses per package). In addition to these baseline environments, we also conducted some boundary experiments to attempt to 'stress' the SMR and Combo mechanisms. These 'stressful' environments employed 6 subjects, one or more of whom had one large all encompassing package whose value was close to, but not as great as (we call this the "gain"--high or low), that in the optimal allocation which required a diversified set of winners of individual licenses. In an additional treatment of the 'stressful' environment, we gave the individual with the large package one of the smaller packages of the winning allocation to see if he could "bump" himself when it was in his own interest (we call this the "own" feature) to do so.

We compared the results of the version of the Combo auction tested to the most efficient version of the SMR auction tested in which all licenses had equal points and flexibility was constant at 1.5 throughout the auction. A recent Congressional mandate requires that the FCC consider assignment efficiency as the primary measure of performance when evaluating the desirability of using a particular auction mechanism for assigning spectrum licenses. A secondary factor to be considered is the revenue that will be generated by the auction. In addition to these two criteria, we also measured the length of time, expressed in rounds, the auction took to close, and whether winning bidders incurred substantial losses from acquiring their licenses. In the three tables below we list the mean value of the observations for each environment/mechanism pair.

EFFICENCY

Environment →	**Additive**	**Low**	**Medium**	**High**
Mechanism				
SMR	97%	90%	82%	79%
Combo	99%	96%	98%	96%

Result: The Combo was more efficient than the SMR in all environments, with biggest difference coming in high synergy case.

REVENUE

Environment →	**Additive**	**Low**	**Medium**	**High**
Mechanism				
SMR	4631	8538	5333	5687
Combo	4205	8059	4603	4874

Result: The SMR generated higher revenues than Combo did in all environments, with biggest difference coming in the high synergy case. This was due to a tendency for SMR winning bidders to incur substantial losses from acquiring licenses after failed package aggregation.

ROUNDS

Environment →	**Additive**	**Low**	**Medium**	**High**
Mechanism				
SMR	8.3	10	10.5	9.5
Combo	25.9	28	32.3	31.8

Result: The Combo mechanism took about 3 times longer to finish than the SMR.

We conducted 18 Combo auctions with the 'stressful' parameters described above, and 10 SMR auctions with those same parameters. We had three synergy environments (Additive, Medium and High), and three treatments (Own_high_gain, Own_low_gain, high_gain). The table below supplies the averages (efficiency, revenue, rounds).

	SMR	**Combo**
Additive		
Own_high_gain	(100, 4035, 6)	(93,5102,40)
high_gain	(94, 5803, 17)	(94,5686,31.5)
Own_low_gain	(100, 4335, 8)	(95.5, 4470, 20.5)
Medium		
Own_high_gain	(100, 3124, 6)	(99, 4421, 44.5)
high_gain	(92, 4205, 17)	(95, 4106, 31.5)
Own_low_gain	(99, 3185, 8)	(94.5, 3500, 20.5)
High		
Own_high_gain	(59, 3082, 15)	(78.5, 2925,29)
high_gain	(63, 3223, 10)	(88,3400, 45)
Own_low_gain	(70, 2496, 18)	(100, 3270, 42)

Result: In all cases the Combo auction again took two to three times as many rounds to complete. In the additive case (no synergies) the SMR was slightly more efficient than the Combo was, while this result dramatically reversed in the high synergy environment.

There is clearly a tradeoff to be made between the higher efficiency of the combinatorial auctions and the transaction cost of its extended duration. There may be better implementations for the Combo auction which reduce its expected duration while continuing to deliver high efficiency. The stress tests conducted provide some indication of interaction between value environment, which may be impossible to know well in a real implementation, and choice of optimal institutional rules.

1.0 Introduction

The Federal Communications Commission ("FCC") has assigned Cybernomics Inc. the task of comparing the performance properties of the auction it currently uses to assign radio spectrum licenses (herein referred to as the Simultaneous Multiple Round ("SMR") auction), and a new auction form in which bidders are able to submit bids for single licenses as well as bids for combinations of such licenses (a so-called "combinatorial" auction). In a companion report (Cybernomics (1999)) the performance of the SMR was analyzed in a variety of economic environments. In this report we examine the performance properties, in the same set of economic environments, of an FCC approved version of the combinatorial auction (Combo) proposed by CRA (1999) as part of its Task 1 contract with the FCC. This institutional comparison is designed to provide data on the operation and implementation of a new allocation process for spectrum licenses. We begin by describing, in detail, the mechanisms we tested in our economic environments.

2.0 The Auction Mechanisms

Each of the auctions analyzed, SMR and Combo, had many parameters to be selected. We opted for the most transparent sets of rules, from the participants' points of view, in streamlining both auction processes. In the case of the SMR auction, from our previous study (Cybernomics (1999)) we found that having equal eligibility points for each license had a positive effect on efficiency and revenue. In addition, we did not employ waivers or upfront payments in our design given that the reasons for these rules, e.g. technical issues, firm financing, etc., were not present in our controlled setting. For the Combo auction, in consultation with the FCC and the CRA designers, our experiments employed a streamlined version of their mechanism that focused on assisting bidders in searching for fitting combinations and simplified eligibility constraints as much as possible.

Common across each of the auction forms were ten licenses offered for sale. The licenses were generically labeled A, B, C, ..., J and had values to bidders ranging from no synergy among licenses to a high degree of synergy among licenses. The rules of the market mechanisms are detailed next.

2.1 SMR Auction Rules

Participants submit a series of single-item, sealed bids for desired licenses. Following the submission of such bids, the high bid for each license is posted. These high bids then become the standing bids for the next round of bidding. In addition to these general rules, we adopted the following specific rules:

0. *Activity:* In order to be able to submit a bid in a round, a participant must have submitted an acceptable bid in the previous round or have had a standing bid two rounds previous.

1. *Acceptable bids:* In order for a bid to be *acceptable* in any round, it must be greater, by a pre-specified increment, than the standing bid for that license.

2. *Bid increments:* The minimum acceptable bid for a license in the next round is the current standing bid plus a fixed percentage of that standing bid. The fixed percentage is determined by the excess bids placed on the license. Specifically, if one excess bid was submitted on the license the minimum acceptable bid would be (1+.05)*(standing bid); if two excess bids were submitted on the license then the minimum acceptable bid would be (1+.10)*(standing bid); if three or more excess bids were submitted on the license then the minimum acceptable bid would be (1+.15)*(standing bid).[1]

3. *Bid Withdrawal:* Subjects are allowed to *withdraw* any of their standing bids before a round begins. After a withdrawal, the FCC becomes the standing bidder for the withdrawn license and replaces the bid with one that is less than or equal to the withdrawn bid. An individual who withdraws a bid pays a *penalty* equal to the maximum of zero or the difference between the amount of the bid he withdrew and the highest bid submitted by a participant other than the FCC after

[1] We imposed what the FCC refers to as "click-box" bidding. This form of bidding allows the bidder to only increase its bid in integer multiples of the identified increment. Thus, if the increment amount were 10% for a particular license, any bid submitted for that license was restricted to be Standing Bid * (1 + π*.10), where π is a positive integer greater than or equal to 1.

his withdrawal.[2] In the past, the FCC did not limit the number of withdrawals a participant could use. With concurrence from the FCC, we used a limit of 2 rounds of withdrawals per subject.[3]

4. *Eligibility:* Each license has the same fixed number of eligibility points associated with it, namely 1. A participant is constrained to bid for at most a subset S of licenses, other than the ones she has the current standing bid on, that satisfies the following eligibility constraint:

   $(\sum_{i \in S} \text{Points}_i) \leq$ (the sum of the eligibility points of licenses for which the participant submitted acceptable bids in the previous round and does not currently have the standing bid + the sum of the eligibility points of licenses for which the participant had the standing bid 2 rounds previous but no longer has the standing bid)

5. *Stopping rule:* The auction stops when all bidders have no eligibility remaining beyond their standing bids. The items are awarded to the participants with the standing bids and any withdrawal penalties are paid at that time.

6. *Information:* All bids submitted by all participants are revealed after every round.

2.2 Task 1 Combinatorial Auction

As with the SMR, the combinatorial auction proceeds in rounds. In each round, participants can submit a series of sealed single-item and/or packaged bids.[4] Following the submission of such bids, the largest sum of the submitted bids, such that each license is allocated to only one participant and the package constraints are not violated, is posted for all to see along with the best single license bids. All of the bids submitted in the current round are used to "constrain" the bids that can be submitted in the next round. The constraints are determined using the following specific rules.

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[2] Because a standing bid on a license may be withdrawn multiple times, the highest bid after a withdrawal need not be the final bid on a license.

[3] Also, with agreement from the FCC, we combined the bidding and withdrawal phases of the auction. While this speeds up the auction, it does create an uncertainty on the part of the withdrawing party who cannot signal the availability of a license before bidding begins, which may invite new bids reducing the withdrawal penalty.

[4] A package bid is defined as a bid for two or more licenses together with a single bid in which all licenses in the package must be accepted at the single bid or none should be accepted.

0. *Activity:* In order to be able to submit a bid in a round, a participant must have submitted an acceptable bid in the previous round or have had a standing bid two rounds previous.
1. *Acceptable bids:* In order for a bid to be *acceptable* in any round, it must be 5% greater than the best combination of bids, from the previous round, which exactly span the licenses in the bid.
2. *Bid Cancellation:* Bids from previous rounds, whether they are winning bids or not, are automatically submitted in the next round and count against eligibility unless they are removed or canceled from the system.[5]
3. *Eligibility:* Each license has the same fixed number of eligibility points associated with it, namely 1. Participants cannot bid for more items than they have eligibility points. Eligibility has two rules. The first rule is that the union of all bids (that is the number of distinct licenses bid on whether they are in packages or singles) cannot exceed a bidder's eligibility. The second rule for is that the sum of the number of items in packages (for all packages of two items or greater) cannot exceed a bidder's eligibility. Thus, if a participant has 5 eligibility points, he can bid on any five single licenses AND also place package bids as long as the number of items in all packages submitted is less than or equal to five. Eligibility in the next period is the minimum of the participant's eligibility in the previous round or the activity in the current round which is defined by:

   $\lambda \cdot (\beta_1 \cdot$*[number of licenses you are currently winning]* + $\beta_2 \cdot$*[number of licenses in bids that are not winners but meet or exceed the minimum increment]* + $\beta_3 \cdot$*[the number of licenses that are not in the winning set or meet the minimum increment but are above the best single item bids]*

   In our experiments we set $\lambda=3$; and $\beta_1=\beta_2=\beta_3=1$.
4. *Stopping rule:* The auction stops when no new acceptable bids are submitted.[6]

[5] The reason for leaving old bids or bids that do not meet the minimum is to allow bidders to fit together even though their bids alone do not win.

[6] Even though new bids are submitted it does not mean that revenue has increased. This occurs because an acceptable bid must be better than the best bids in the system the bid replaces, not the winning bids.

5. *Information:* All bids submitted or left in the system by all other participants are revealed after every round.

3.0 Experimental Design

We begin by describing the various valuation structures we used to induce the market demand for licenses and then describe the training protocols for subjects. The exact same valuation structures were used for the set of experiments conducted to test the performance of the SMR.

3.1 Market Demand

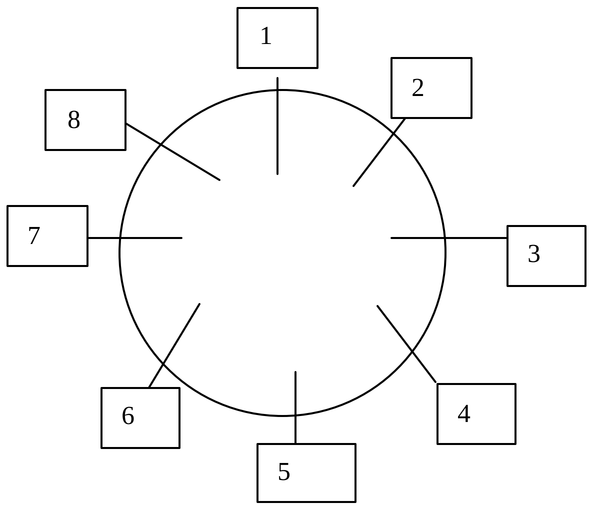
Subject i's valuation ($V^i(X)$) for a subset of licenses (X) is parameterized by the following expression:

$$V^i(X) = \Sigma_{j \in X} V^i_j + \lambda^i (\Sigma_{j \in X} q_j)^{\beta^i} + \Delta^i (\Sigma_{j \in X} \Sigma_{k \in A^j} \delta^j(k))^{\alpha^i}$$

The first term represents the sum of the stand-alone values (V^i_j) of the licenses in the subset X. The next two terms were added to model two potential license value superadditivities. The second term captures a scale economy a bidder achieves when using, for instance, its existing billing service and maintenance departments with the new service.[7] The reduction in the average total service cost per customer introduces a superadditivity in a bidder's license valuations for a given set of licenses (the variable q_j represents the population associated with license j). The third term attempts to capture license value superadditivity that results from the value a mobile service subscriber places on being able to roam seamlessly from one service area to another. Due to this effect, some bidders may experience a superadditivity in license valuation across a set of contiguous licenses (the set A^j represents the set of licenses that are contiguous to j). To represent this type of value superadditivity, consider the topology of licenses described in Figure 3.1 below. Each point on the wheel represents a license and a geographical location. Licenses adjacent to each other (e.g., license {1, 2} and {8,1}) are considered contiguous.

[7] This structure is somewhat similar to the econometric model developed by Ausubel et al. (1997).

Figure 3.1



If we let δ^j denote a zero/one indicator function for license j, then we can describe the contiguity relationship as follows:

$$\delta^j(k) = \begin{cases} 1 & \text{if } k = +/-1 \ j \bmod n; \\ 0 & \text{otherwise.} \end{cases}$$

Experiments were conducted under four different valuation environments: (1) additive; (2) low superadditivity; (3) medium superadditivity; and (4) high superadditivity. The four environments were generated by varying the parameters, λ, β, Δ, and α, the separate increases which lead to an increase in the degree of license value superadditivity.[8] Table 3.3.1 below shows the parameters used in each environment:

[8] See "Testing Combinatorial and Non-Combinatorial Auction Designs: Environment and Testing Protocol Report" prepared for the Federal Communications Commission by Cybernomics Inc., April 12, 1999 (Contract # C-9854019) for a detailed description of the parameter values.

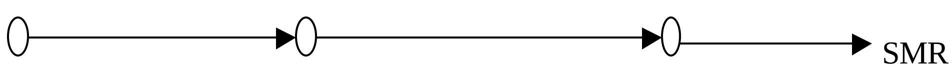
Table 3.1.1

Environment	λ	β	Δ	α
High	175	1	230	2.05
Medium	150	1	229	1.65
Low	78	.65	120	1.65

3.2 Subject Pool and Procedures

A total of 150 subjects were recruited during the 1999 Spring semester from upper-class courses in accounting, information systems, economics and engineering at the University of Arizona to participate in the SMR experiments. Each subject was trained in the auction rules through a series of 3 two-hour experimental sessions in which each participated in a progression of more complicated SMR rules. Figure 3.2 lists the training progression, and associated treatments, experienced by various subjects. Subjects were considered trained after completing the series of training sessions.

Figure 3.2 SMR Subject Training Protocol



Additive Values	Additive Values	Synergies
No Withdrawal	Withdrawal	Withdrawals
No Activity Rules	Activity Rules	Activity Rules
Increment Bidding	Increment Bidding	Increment Bidding

As shown above, each subject participated in a series of SMR auctions in which withdrawal rules, activity rules and synergy values were added sequentially to the mechanism and environment

In the same manner we recruited 100 subjects from upper-class courses in accounting, information systems, economics and engineering at the University of Arizona during the Fall 1999 semester. Subjects participated in two 2 and 1/2 hour experimental sessions in which they faced participated in a progression of more complicated Combinatorial auction rules.[9] Subjects could not participate in the full combinatorial auction until they had complete the 5 hours of training.

3.3 Design Summary and Procedures

Table 3.6 lists the conducted experiments. An experiment includes a series of SMR or Combinatorial auctions. Subjects are assigned different valuation across the auctions. Each auction involved the sale of ten licenses (i.e., A through J). Superadditive valuations applied to two discrete license sets; Set Φ = (A, B, C, D, and E) and Set Ψ = (F, G, H, I and J).[10] Each set was assigned either an additive, or low, medium, or high superadditive value environment. For example, in the first experiment involving the Baseline treatment Set Φ had an additive value environment while Set Ψ had a high superadditive environment. Instructions for each of the treatments can be found at http://linus.econlab.arizona.edu/FCC_Instructions, the experimental parameters used can be found at http://linus.econlab.arizona.edu/FCC_Parameters and the data for each experiment can be found at http://linus.econlab.arizona.edu/FCC_data.[11]

Table 3.6 Experimental Design

Treatment	**Experiment**	**Date**	**Environments (Φ,Ψ)**
SMR	1	4/16/99	Additive, Medium
SMR	1	4/16/99	Low, High
SMR	1	4/16/99	High, Additive

[9] The experiments were extended to 2 and 1/2 hour when we found that the experiments were taking much longer to finish than the simpler SMR experiments.
[10] For example, the set of licenses that generate synergy values do not contain elements from sets Φand Ψ.
[11] In this report, we do not use the experiments from the treatments in Cybernomics (1999) in which licenses had the same number of eligibility points.

SMR	2	4/21/99	Additive, Medium
SMR	2	4/21/99	Low, High
SMR	2	4/21/99	High, Additive
SMR	3	4/27/99	Low, High
SMR	4	5/3/99	Medium, Low
SMR	4	5/3/99	Medium, Low
SMR	4	5/3/99	Low, High
SMR	4	5/3/99	Medium, Low
SMR	5	5/4/99	Medium, Low
SMR	5	5/4/99	Low, High
SMR	5	5/4/99	High, Additive
SMR	6	5/7/99	Additive, Medium
SMR	6	5/7/99	High, Additive
SMR	6	5/7/99	Low, High
SMR	7	8/3/99	Low, High
SMR	8	4/27/99	Medium, Low
SMR	9	4/29/99	Additive, Medium
SMR	9	4/29/99	High, Additive
SMR	9	4/29/99	Additive, High
SMR	10	4/30/99	Additive, Medium
SMR	10	4/30/99	Low, High
SMR	10	4/30/99	High, Additive
SMR	11	5/3/99	Additive, High
SMR	11	5/3/99	Medium, Low
SMR	11	5/3/99	Medium, Low
SMR	11	5/3/99	High, Additive
SMR	12	5/4/99	Additive, Medium
SMR	12	5/4/99	Medium, Low
SMR	12	5/4/99	Medium, Low
SMR	12	5/4/99	High, Additive
SMR	13	5/7/99	High, Additive
SMR	13	5/7/99	Medium, Low
SMR	14	8/3/99	Low, High
Combinatorial	1	10/2/99	Medium, Low
Combinatorial	2	11/2/99	Medium, Low
Combinatorial	2	11/2/99	Medium, Low
Combinatorial	3	11/5/99	High, Additive
Combinatorial	3	11/5/99	Additive, High
Combinatorial	4	11/8/99	Low, High
Combinatorial	4	11/8/99	Additive, Medium
Combinatorial	5	11/9/99	Medium, Low
Combinatorial	5	11/9/99	Additive, High
Combinatorial	6	11/10/99	Low, High
Combinatorial	6	11/10/99	Additive, Medium
Combinatorial	7	11/11/99	Low, High

Combinatorial	7	11/11/99	High, Additive
Combinatorial	7	11/11/99	Low, High
Combinatorial	8	11/12/99	High, Additive
Combinatorial	8	11/12/99	Medium, Low
Combinatorial	9	11/15/99	Low, High
Combinatorial	9	11/15/99	Additive, High
Combinatorial	10	11/16/99	Medium, Low
Combinatorial	10	11/16/99	Additive, Medium
Combinatorial	11	11/19/99	Medium, Low
Combinatorial	11	11/19/99	Additive, High
Combinatorial	12	11/22/99	High, Additive
Combinatorial	13	11/23/99	High, Additive

4.0 Comparative Experimental Results

A recent Congressional mandate requires that the FCC consider assignment efficiency as the primary measure of performance when evaluating the desirability of using a particular auction mechanism for assigning spectrum licenses. A secondary factor to be considered is the revenue that will be generated by the auction. In addition to these two criteria, we also measured the length of time, expressed in rounds, the auction took to close, and whether winning bidders incurred substantial losses from acquiring their licenses. Auction duration and bidder loss measures have important practical significance. Auctions that take a long time to close impose a heavy transaction cost upon participants and the FCC. Similarly, heavy losses by bidders may result in the re-auctioning of the acquired licenses. The re-auctioning of the license extends the period of license non-use and, in so doing, deprives society of important benefits. None of these difficult-to-measure transaction costs are part of the standard measure of allocational efficiency defined below. Finally, we examine the frequency with which participants withdraw high bids. In some environments, a high number of withdrawn bids indicate that bidders are having substantial difficulty in obtaining their most desired licenses.

4.1 Efficiency

An auction is said to be 100% efficient when it assigns the set of offered items so that the total value that society obtains from the items is maximized. The efficient set of assignments is known as the optimal allocation. An auction mechanism's ability to efficiently assign items is measured as the ratio of the sum of the values that assignees place on their items divided by the sum of the value maximizing assignment. Thus, efficiency losses occur when some bidder fails to make a purchase that is in the optimal allocation, or some other bidder makes a purchase that is not in the optimal allocation.

4.1.1 Additive Environment

Table 4.1.1a shows the average (and median) level of assignment efficiency of the SMR and Combo auctions for the additive environment. Table 4.1.1b shows the number of 100% outcomes from the total number of experiments conducted.

Table 4.1.1a Mean (Median) Efficiency by Auction: Additive Environment

	% Efficiency
SMR	96.6 (100)
Combo	99.2 (100)

Table 4.1.1b Count of 100% Efficiency Outcomes by Auction Additive Environment (n = number of total experiments)

	100% Efficient Outcomes
SMR	11 (n=17)
Combo	10 (n=11)

<u>Result 1</u>**:** *The Combo auction is slightly more efficient than the SMR in the additive environment but both generate very high efficiencies.*

Table 4.1.1a shows a mean difference of *2.6%* with a rank sum test statistic of Z=1.42 which is significant at the 10% level of confidence. In our report, Cybernomics (1999), we found that having more flexibility in eligibility increases efficiency. Thus, we conjecture that the difference in efficiency stems from the fact that the Combo auction is more generous in its flexibility in that it uses a rule of 3 times activity to determine eligibility. We also point out that although bidders are free to submit packages in the additive environment they rarely do so. Fewer than 2% of all bids are package bids in the additive environment.

4.1.2 Superadditive Environment

We conducted experiments with various levels of superadditivity – low, medium, and high – associated with each subgroup of 5 licenses. Table 4.1.2a provides the mean (median) for each superadditive environment.

Table 4.1.2a Mean (Median) Efficiency by Treatment: Superadditive Environments

	Low	**Medium**	**High**
SMR	80.6 (82.5)	79.0 (92.0)	77.1 (84.0)
Combo	96.2 (99.0)	96.1 (100)	98.0 (100)

***Result 2**: In every superadditive environment the Combo auction outperforms the SMR with average efficiency gains ranging from 16% - 22%. The largest gain occurs in the high synergy case.*

The rank sum statistics for each superadditive environment is Z=9.76 (low), Z=8.75 (medium), Z=12.36(high) which are significant at less than 1%. In Figures 4.1.2.a-c we supply the distributions of efficiencies for each superadditive value environment.

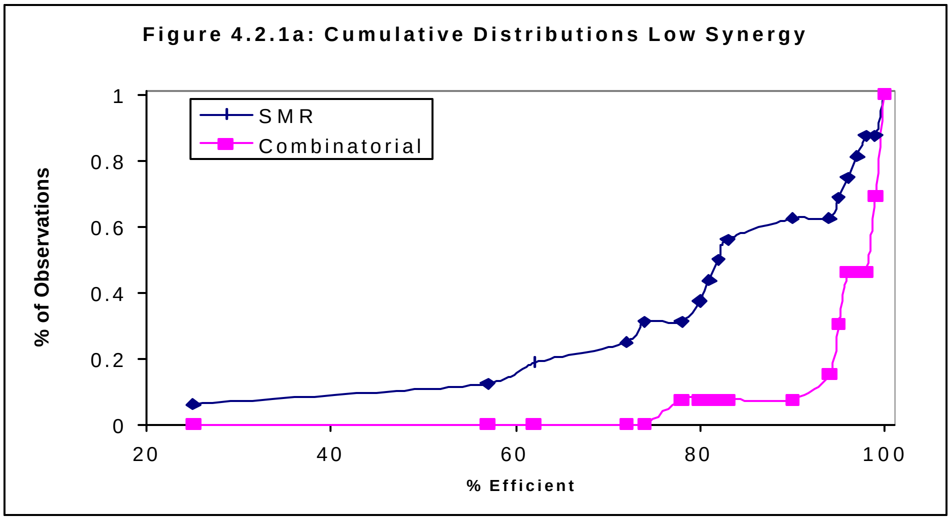


Figure 4.2.1a: Cumulative Distributions Low Synergy
SMR
Combinatorial
% of Observations
% Efficient
0
0.2
0.4
0.6
0.8
1
20
40
60
80
100

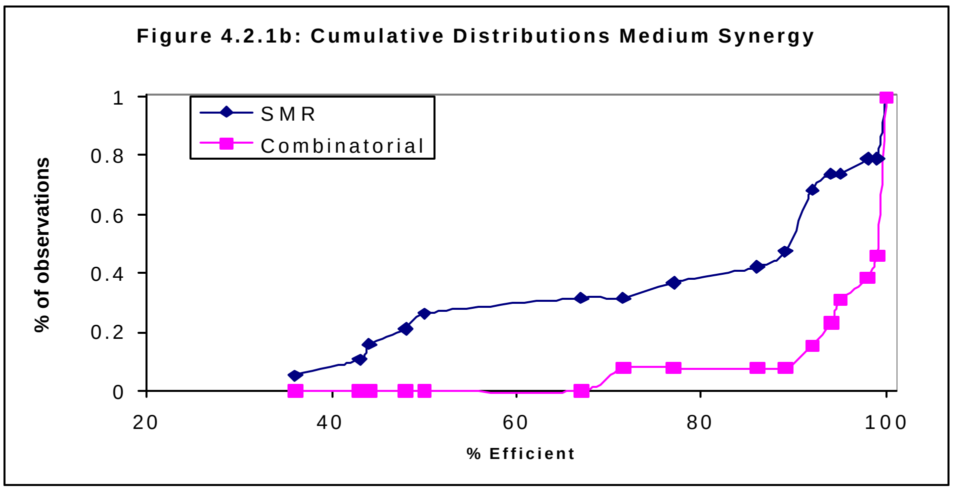


Figure 4.2.1b: Cumulative Distributions Medium Synergy
SMR
Combinatorial
% of observations
% Efficient
0
0.2
0.4
0.6
0.8
1
20
40
60
80
100

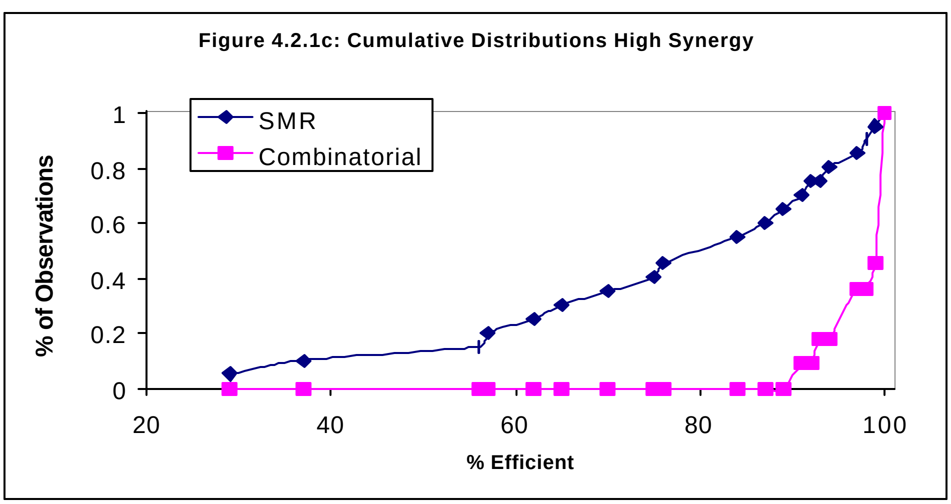


Figure 4.2.1c: Cumulative Distributions High Synergy
SMR
Combinatorial
% of Observations
% Efficient
0
0.2
0.4
0.6
0.8
1
20
40
60
80
100

4.2 Revenue

The amount of revenue generated from a particular auction mechanism depends, in part, on the distribution of license valuations across active bidders. This distribution is altered by changes in the number of bidders in the auction: for example, when financial difficulties occur and a bidder decides or is forced to leave. One method for controlling the confounding effect of changes in the number of bidders on auction revenue is simply to "normalize" auction revenue on the basis of what could be realized given the set of remaining participating bidders and their valuations.

4.2.1 Additive Environment

In the additive environment analysis, we normalize "revenue" as the ratio of revenue actually collected divided by the sum of the competitive prices for the remaining agents.

Table 4.2.1a Normalized Mean (Median) Revenue: Additive Environment

	Revenue
SMR	1.36 (1.38)
Combo	1.28 (1.26)

Result 3. *Revenue is significantly above the competitive equilibrium predictions. The Revenues are higher in the SMR than the Combo auction.*

There is a significant amount of jump bidding (above minimum increment requirements) in both the SMR and Combo auctions, that causes the revenue to exceed competitive expectations. One reason that revenue may be even higher in the SMR is that bids had to be in percentage jumps while in the Combo auction they had to be any number above 5% of the standing bid.

4.2.2 Superadditive Environment

In the superadditive environment analysis, we normalize "revenue" as the ratio of the maximum total value assignment for the participating agents divided by the revenue actually collected. Table 4.3.1a shows the mean (median) normalized revenue ratios:

Table 4.2.2 Normalized Mean (Median) Revenue: Superadditive Environment

	Low	**Medium**	**High**
SMR	.85 (.83)	.95 (.92)	1.05 (1.00)
Combo	.77 (.81)	.89 (.91)	.90 (.91)

Result 4: *In the Superadditive environments, the SMR generates more revenue than the Combo auction (with the largest difference coming in the high synergy case).*

The rank sum statistics for these ratios are Z=3.98 (low); Z=4.63 (medium); Z=8.78 (high), which are highly significant. This result coupled with the efficiency results may seem to be a puzzle. That is, how can the SMR, which is less efficient than the Combo auction, generate significantly more revenue? This is somewhat surprising given similar rules in both auctions. The main reason for the divergence is that in the SMR bidders can lose money because of failed license aggregations. In particular, there are no losses by bidders in the Combo auction, while in the SMR, the losses ranged from 4% to 35% of the revenue raised. These external losses are not part of measured efficiency[12] which simply compares the total social value of realized and optimal allocations, and is not concerned with participant and auctioneer division of surplus.

4.3 Bidding Rounds

One proposed factor in evaluating auctions is their duration. A reasonable assumption is that longer auctions should be avoided *ceteris paribus*. This reduces the transaction costs faced by the bidders and the auctioneer, and thus potentially raises effective valuations and net revenues. In consultation with the FCC and CRA designers the amount of time allowed for bidding in each round was increased for the Combo versus the SMR auction due to the complex nature and potentially large number of bids that the participants might tender in the former. We list the mean and median rounds required to complete each auction in Table 4.3.

Table 4.3 Average Rounds per Auction Type

	Rounds
SMR	8.9 (9)
Combo	29.3 (30)

<u>Result 5:</u> *The Combo auction takes over 3 times as long as the SMR to finish.*

It should be noted, however, that more than 60% of the Combo auctions went for as many as 20 rounds without the realized allocation or revenue changing before the auction closed. There is clearly some opportunity to create an implementation that would curtail this wheel spinning.

In summary, these baseline experiments indicate that when the Combo auction is used, efficiency is increased between 2.6% and 22%, but auction duration is extended considerably.

4.4 Boundary Experiments

Given the success of the CRA Combo auction in generating high efficiencies in the baseline superadditive environments originally tested, we created several additional environments that we thought might put pressure on both auction mechanisms. Such “stress tests” have become a fairly standard part of experimental methodology, particularly in economic design problems. Clients often want to see a challenge to the edges of validity for the principle findings.   Following the examples found in CRA (1998), we focused on the threshold problem presented in their report. In particular, we developed two measures that might influence the ability of the CRA Combo auction to overcome the threshold problem. Recall that the threshold problem occurs when a large package of licenses bid on by a single bidder must be displaced by a group of ‘small’ bidders bidding on subsets of that large package. The ‘small’ bidders must then come to

[12] We could measure efficiency to include external reductions of working capital.

some agreement on how much each will contribute to overcome the large bid in order to split the gains.

4.4.1 Boundary Case Treatments

The experimental treatments devised were based upon the following measures:

Measure 1: The Gain Effect. This effect is one that describes the difference between the value of the optimal allocation (V^*), which is composed of several bidder packages, and the next highest value allocation (~~V~~), which is constructed to be a single bidder's value for a large package covering the optimal set of packages. We define the *Gain* (G) as the ratio G = (V^*)/(~~V~~). As G increases we say the gain is *increased*.

***Measure 2**: The Own Effect.* This effect is one that is coupled with the gain effect. It occurs when j is the large package bidder who demands ~~V,~~ and j is also one of the small package bidders included in V^*. To achieve the optimal allocation j must forego his large package to be included in the optimal allocation of smaller winning packages. J may not collaborate for two reasons: 1, he is in a stronger negotiating position and may want more of the surplus than other small package bidders; or 2, displacing himself, even if it is apparently profitable, may create unpredictable dynamics in the subsequent bidding.

Figures 4.4.1a/b show the various boundary environments we tested using ten licenses (A through J). Actual parameters for these experiments can be found at http://linus.econlab.arizona.edu/FCC_Parameters In Figure 4.4.1a we see that bidder 6 has the large package and is either in the winning set or not with package F,G (own effect). The Gain is either **high** .81 (350/430) or **low** .70 (301/430).

Figure 4.4.1a Gain and Own Effect Case1

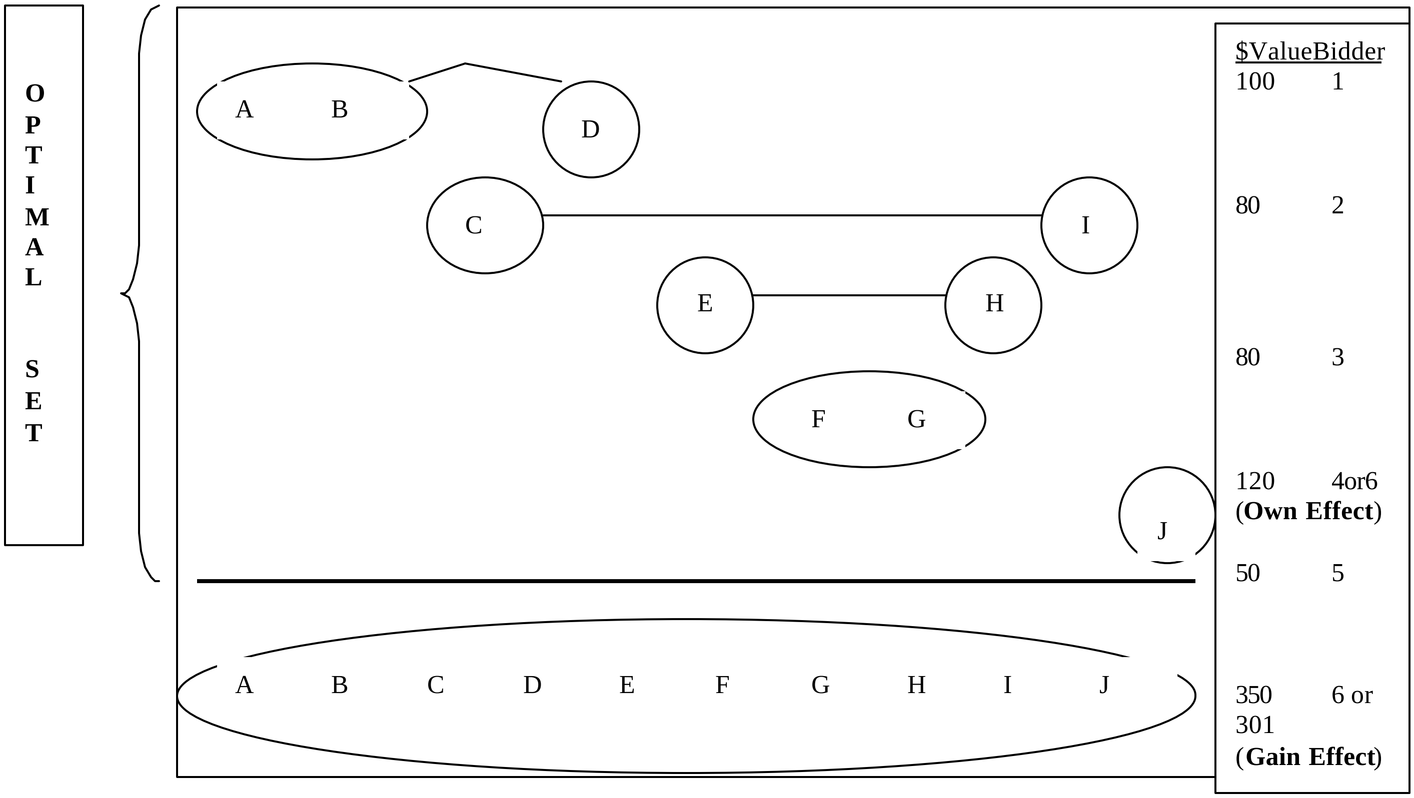
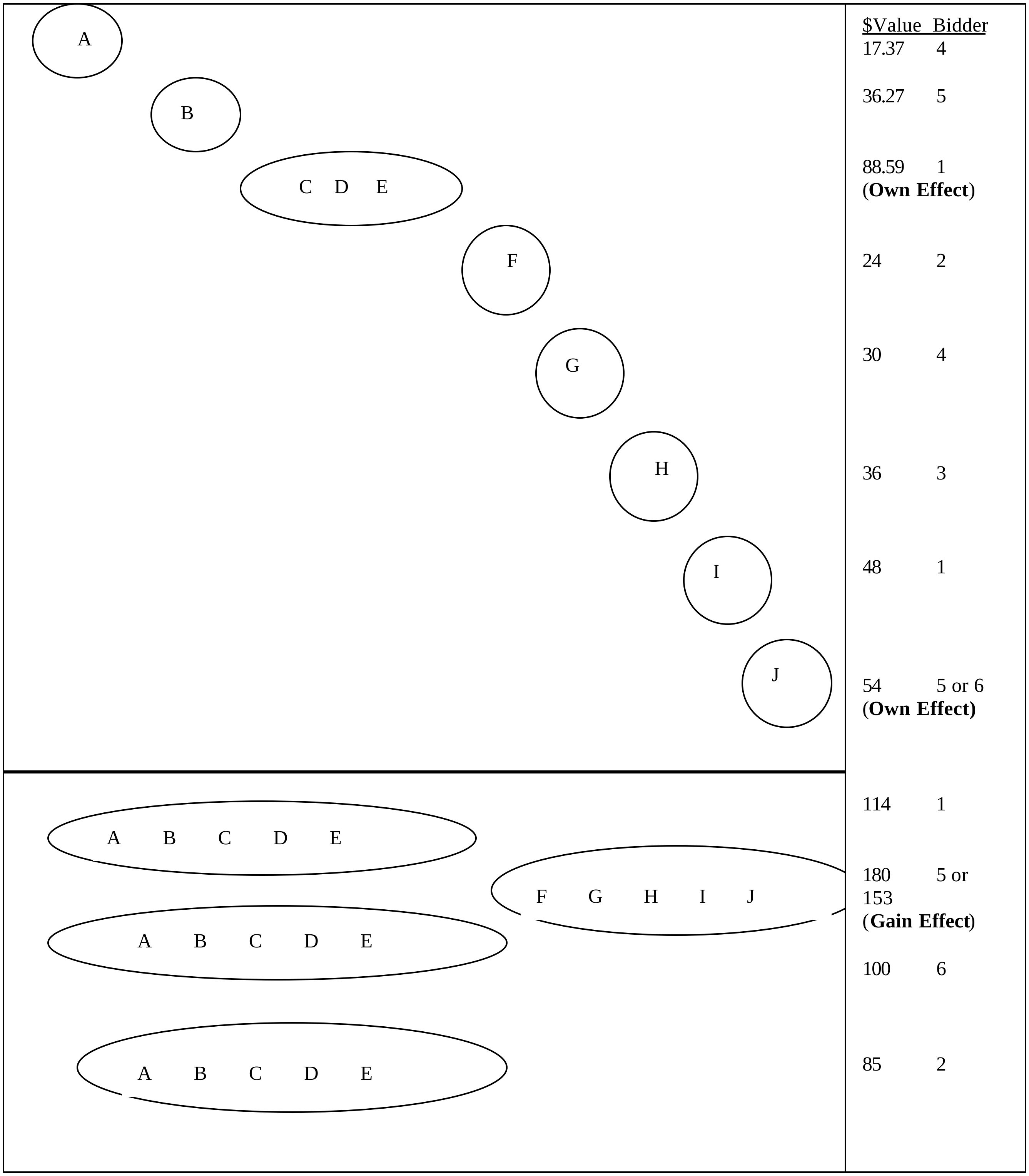


Figure 4.4.1b Gain and Own Effect Case2



In Figure 4.4.1b we divided the licenses into two groups with various own and gain effects. For licenses A-E we had several bidders with values for the entire license group from A to E, while some of them had values for packages which were part of the optimal allocation set. In the second group of licenses F-J, each bidder has values for single licenses except bidder 5 who has value for the entire group of licenses F-J. We divide these two groups into Cases 2a and 2b respectively.

Table 4.4.1c shows our experimental design for these boundary case treatments.

Table 4.4.1c Experimental Design Boundary Cases

Case	Auction	Gain	Own	# of Auctions
Case 1	Combo	.81	Yes	3
Case 1	Combo	.81	No	3
Case 1	Combo	.70	Yes	2
Case 1	SMR	.81	Yes	1
Case 1	SMR	.81	No	1
Case 1	SMR	.70	Yes	1
Case 2a	Combo	.94	Yes	3
Case 2a	Combo	.94	No	2
Case 2a	Combo	.80	Yes	2
Case 2a	SMR	.94	Yes	1
Case 2a	SMR	.94	No	1
Case 2a	SMR	.80	Yes	1
Case 2b	Combo	.80	Yes	3
Case 2b	Combo	.80	Yes	2
Case 2b	Combo	.80	Yes	2
Case 2b	SMR	.80	Yes	1
Case 2b	SMR	.80	Yes	1
Case 2b	SMR	.80	Yes	1

4.4.2 Boundary Case Results

Table 4.4.2 shows the efficiencies for each experiment we conducted.

Table 4.4.2: Efficiency Results for Boundary Cases

Case	Auction	Gain	Own	Efficiencies %
1	Combo	.81	Yes	78, 79, 78
	SMR			59
	Combo	.	No	97, 79
	SMR			63
	Combo	.70	Yes	100, 100
	SMR			70
2a	Combo	.80	Yes	99, 99, 99, 95, 94, 95, 95
	SMR			100, 99, 95, 95
2b	Combo	.94	Yes	91, 94, 94
	SMR			100
	Combo		No	95, 95
	SMR			100
	Combo	.80	Yes	100, 91
	SMR			100

Result 7*: As the gain is increased, efficiency falls. The own effect also reduces efficiency. Even though efficiencies are low for the Combo auction in Case 1, they are higher than for the SMR. In Case 2b in which bidders only have values for single license packages, except for one bidder who has a value for all the licenses, the SMR outperforms the CRA Combo auction.*

While there seems to be a gain and own effect, we notice that in Case2a in which the gain is "high" and there is an own effect, efficiencies are relatively high and the same across auction forms. This suggests that the impact of the gain and own effects are interrelated with the constellation of packages. Thus, the environment and rule composition of the institutions tested interacted to determine efficiency. To choose between the versions of the SMR and CRA Combo tested, one might need a priori knowledge of the environment, which may be impossible to obtain.

4.5 Implementation and Strategic Issues

One design parameter for which there was no guidance for us in our implementation of the CRA Combo auction was the amount of initial eligibility to be awarded each participant. Setting initial eligibility just equal to the number of licenses available can severely restrict the number of packages that one can simultaneously submit. This would hamper the ability of bidders to find fits and for the optimization algorithm to do its work. This seems to be a very crucial parameter that, as far as we can surmise, is not specified in any of the documentation of the CRA Combo auction. After some preliminary trials, we set the initial eligibility at 30 points, which allowed the bidders the possibility to express an opening interest in all packages that that were of value to them.

One strategic ploy we observed in several of the auctions is manifested by the following bidding tactic. Bidder i would place a winning bid on a large package, say ABCD, and also place a high bid on the subset package AB for which he had a little value, but which was being pursued by another bidder j. In the next round, to remain eligible bidder j would be forced to bid above the AB bid placed in the previous period by bidder i, though i was free to immediately and unannounced remove that bid.[13] This lack of required commitment on previous round non-winning bids can provide poor information for competitive bidders, affecting eligibility constraints and auction efficiency.

[13] For example suppose i has a value of 800 for ABCD while j has a value of 500 for AB and k has a value of 500 for CD. Suppose there are other items that j and k have value for, but in the optimal allocation they should combine to win AB and CD. Suppose j and k currently hold the winning bids of 50 for AB and CD respectively. Employing the above technique, i can bid 600 for ABCD and 500 for AB. Bidder j will be turned away to spend his eligibility on other more affordable licenses, while i simply removes his AB bid immediately to prevent k an opportunity to combine with it.

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