A General
Combinatorial Auction Procedure

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We describe a general combinatorial auction procedure called PAUSE (Progressive Adaptive User Selection Environment) which incorporates synergies by permitting all combinatorial bids, is transparent to the bidders, and is computationally tractable for the auctioneer and thus very efficient to run. The inherent computational complexity of combinatorial bidding cannot be eliminated. However, in this auction the computational burden of evaluating synergies rests with the bidders claiming those synergies, while the auctioneer simply checks that a bid is valid. Although the combinatorial bidding mechanism is precisely specified, the specific parameters and auxiliary rules of the auction need to be determined; we make suggestions for future work in this direction. Also included is an example that illustrates how the procedure could be used to assign radio spectrum licenses.

(*Auctions; Combinatorial Bidding; Spectrum; Telecommunications; Synergies*)

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1. Introduction

It is not difficult to think of examples of auctions in which the value of a property to a bidder is increased if another property or group of properties is won by that bidder; this superadditive or synergistic effect may be bidder-specific. Auction authorities clearly have an incentive to structure their auctions so as to allow bidders to realize their synergies on combinations of properties in such a way that will be both fair to the bidders and practical to implement. The most obvious approach is to permit bids on groups of properties, called *combinatorial bids*.

Without the allowance of combinatorial bids, bidders will face "exposure risk" (Rothkopf, Pekeč and Harstad 1998). Suppose that an individual bidder has a synergy that is specific to him on a particular block of properties. He may find that an unsuccessful attempt to acquire the block leads him to commit to a price for a group of properties that is higher than what they are worth to him. Alternatively, he may be unwilling to risk bidding above the sum of his individual valuations, and thus may not be able to obtain the block for which the synergy makes him the efficient recipient.

Given the considerable importance of combinatorial bids, it may be surprising that few auctions—in either theory or practice—have allowed combinatorial bidding. This is undoubtedly due to the fact that combinatorial bidding is computationally burdensome. For example, Rassenti, Smith and Bulfin (1982) developed a combinatorial auction procedure for airport time slots to competing airlines. (See also Grether, Isaac and Plott (1989) and McCabe, Rassenti and Smith (1991).) Their optimization procedure is based on a mathematical programming formulation; however, the authors themselves point out that even a four-city problem will dictate a problem of enormous dimensions. In general, the auctioneer's problem of determining an optimal set of bids in a combinatorial auction is an NP-complete problem.

1.1 The PCS Auction

In the mid 1980s, the Office of Plans and Policy (OPP) of the Federal Communications Commission (FCC) issued a working paper that was soon to have an enormous impact. The paper, by Evan Kwerel and Alex D. Felker, proposed that FCC licensees be selected not by lottery or through hearings—as had been the standard practice to that point—but rather via auction (Kwerel and Felker 1985). This proposal came to fruition with the passage of the Omnibus Budget Reconciliation Act (1993), whereby Congress authorized the use of auctions for assigning the electromagnetic spectrum for Personal Communications Services (PCS).[1] The OPP was given the responsibility for managing the process, which included developing general spectrum auction rules.

The structure adopted was that of a *simultaneous multiple-round auction*: collections of licenses are auctioned simultaneously during discrete rounds, where no sale takes place until the bidding is concluded on all licenses. The PCS auction proceeds in three stages, each with an unspecified number of bidding rounds. At the end of each round, the highest bid becomes the leading bid, and the results are made available to all bidders before the start of the next round. At the end of the last round of the third stage, the leading bidder on each property is designated the sole winner on that property. Detailed rules of the auction can be found in FCC (1993).[2] Key aspects include:

Activity Rules. *Eligibility Requirements.* Each round, a bidder is designated *active* on a particular property if either he has the leading bid from the previous round or has submitted an acceptable improving bid in the current round. In Stage i, bidders are required to remain active on licenses covering a population which is at least A_i percent of the total population for which they wish to remain eligible to bid, where $A_3 > A_2 > A_1$. These eligibility requirements are intended to thwart the "deception effect," whereby a firm might bid cautiously, waiting to see how the others bid while not revealing its own interests until late in the auction.

Bid Increments. In order to be acceptable, a bid must improve the previous leading bid by at least the specified minimum amount (e.g., 5% of the leading bid) set by the auction authority. This helps maintain the speed of the auction.

Transition Between Stages. The auction moves from the first stage to the second when there are no bids on more than T_1 percent of the population base for three consecutive rounds, and from the second stage to the third when there are bids on no more

[1]Personal Communications Services are a broad family of mobile communications services that allow people access to the Public Switched Telephone Network (PSTN) regardless of where they are located. PCS includes not only POTS ("Plain Old Telephone Service") but also data, facsimile, video communication, and other services.

[2]The PCS auction was developed through the efforts of many people. Rothkopf, Pekeč and Harstad (1998) provide complete details.

than T_2 percent of the population base (e.g., values of 5% have been used). The auction closes when bidding stops on every license.

Bid Waivers. The activity rules are balanced by an allocation of a small number of bid waivers to each player to be used at will to maintain eligibility for a round without meeting the eligibility criteria. Bid waivers may be viewed as an effort to increase bidder flexibility.

Bid Withdrawals. A leading bidder is permitted to withdraw his bid during the course of the auction, but is penalized by being required to pay the difference between his bid and the price for which the license is ultimately sold; a winning bidder withdrawing after the close of the auction suffers an extra penalty. Bid withdrawals may be viewed as an effort to reduce the exposure risk to bidders attempting to realize their synergies. The FCC has not yet permitted combinatorial bids in the spectrum auctions. Rothkopf, Pekeč and Harstad (1998) persuasively argue that the disallowal of combinatorial bids is a consequence of the economists' briefs—in particular that of McAfee (1993)—that argued that the only choice was between completely disallowing, or permitting all possible, combinatorial bids, and the latter option in the worst-case scenario would be computationally intractable.

1.2 Computationally Tractable Auctions

Attempts to make the combinatorial auction design problem tractable through specific restrictions on the bidding mechanism have taken the approach of considering specialized structures that are amenable to analysis. Rothkopf, Pekeč and Harstad (1998) discuss these approaches, and ask: What are the least restrictive structures that would result in a computationally tractable problem for the auctioneer to determine the revenue-maximizing outcome?[3] They consider several different structures and constructively show that those they have considered are indeed computationally tractable.

While this manner of dealing with the computational intractability violates what

[3]Here "computationally tractable" is the standard concept that an upper bound on computation time for the given class of computational problems can be expressed as a polynomial function of the size of the input.

might be termed the "McAfee principle" of allowing *all if any* combinatorial bids, Rothkopf, Pekeč and Harstad show in a certain sense how far one can go in this direction: in each instance, they demonstrate their approach to be best possible by proving that the next level of generality will result in an NP-complete problem. For example, they show that allowing bids on arbitrary doubletons reduces to finding a maximal weighted matching in a graph. By the algorithm of Edmonds (1965), such a matching can be found in $O(n^3)$ time. However, they also show that allowing arbitrary tripletons reduces to the 3-set packing problem, which is NP-complete (Karp 1972).

Rothkopf, Pekeč and Harstad do make a convincing case that there are many natural ways to restrict the nature of the bids so as to make combinatorial auctions computationally tractable. However, they also warn that this then raises the problem of deciding *which* combinations of bids to allow.

1.3 Adaptive User Selection Mechanism (AUSM)

Thus far, the only auction approach that has allowed for all possible combinatorial bids is the Adaptive User Selection Mechanism (Banks, Ledyard and Porter 1989). In AUSM, bids can be submitted at any time, where bidders may incorporate any unwithdrawn, currently unsuccessful bids into their own bids, with bidding stopping according to some pre-specified stopping rule. (The original rule specified that the auction stops when no new bid is made soon enough after the last bid.) However, this procedure is susceptible to the "threshold problem." Specifically, a group of players who jointly desire a subset of properties may have difficulty coordinating their bids to displace a single bid on those properties. To address this problem, Banks, Ledyard and Porter designed a modification of AUSM that makes use of a "stand-by queue" in which bidders announce to all other bidders via a bulletin board their willingness to pay a certain price for a specific combination of licenses. As described in Bykowsky and Cull (1997, p. 36): "In essence, the stand-by queue serves as a voluntary contribution mechanism in which prospective contributors attempt to move, in a repeated game context, to a mutually desirable equilibrium."

While a stand-by queue can help overcome the threshold problem, it is less effective at dealing with the related "free-rider problem." A group of players who jointly desire a subset of properties may, in principle, be able to coordinate their bidding via a bulletin board, but each player from the group will have an incentive to wait for the others in the group to improve their bids, thus retaining more of the benefit from the joint bid for itself. Cramton (1997) also points out that AUSM weakens a central advantage of auctions, viz., *transparency*. Transparency means that a losing bidder who offered a higher bid for part of a combination should always be able to see why he lost. Unfortunately, this is generally not the case under AUSM.

1.4 Jump Bidding

A potential problem with simultaneous, multiple-round auctions has been reported by McAfee and McMillan (1996). In the portion of the PCS auction known as the MTA auction,[4] aggressive bidding in early rounds took the form of "jump bidding": entering bids far above that required by the minimum bid increment. The intention of this tactic, which we call more specifically *price-jump bidding*, is to warn weaker rivals against competing on specific properties. A combinatorial auction presents the possibility of another type of jump bidding, *block-jump bidding*, in which a bid by a powerful player for a block of several properties could be effective at preventing small players from piecing together a comparable composite bid, i.e., the threshold problem.

Although McAfee and McMillan describe jump bidding as merely a part of bidder strategies, clearly it is of concern to the FCC (1997, paragraph 143):

> Several commentators suggest that jump bidding is not a problem of serious concern. Some theoretical literature, however, suggests that bidders could use jump bidding to manipulate the auction process and potentially reduce efficiency of the auction.

[4] The MTA auction ran from December 1994 to March 1995 and sold broadband licenses covering the 51 "Major Trading Areas," or MTAs, into which the United States is divided.

1.5 Combinatorial Auction Desiderata

Based on the preceding, we can present a list of properties that we would want to see in a combinatorial auction design. The deception effect can be addressed in activity rules along the lines of those used in the PCS auction. In addition, the auction should:

(a) Be transparent to the bidders;

(b) Present the auctioneer with a tractable bid evaluation problem;

(c) Have a bounded completion time; and

(d) Prevent against jump bidding and mitigate the threshold problem.

2. The PAUSE Auction

We describe a general fully combinatorial auction procedure called PAUSE (Progressive Adaptive User Selection Environment). In particular, this procedure can be used as a basic framework for a combinatorial spectrum auction. More specifically, PAUSE is a two-stage procedure, where:

> Stage 1 is a simultaneous, multiple-round auction, with progressive eligibility requirements and an improvement margin requirement, with bidders submitting bids on individual properties; and
>
> Stage 2 is a simultaneous, multiple-round auction with progressive eligibility requirements and an improvement margin requirement, with *composite bids* to facilitate realization of player synergies.

The main contribution of this paper is the combinatorial auction described in Stage 2, and there are several consistent ways that Stage 1 might be organized. The structure of Stage 1 we describe below makes minimal changes from that used in the PCS auction.

PAUSE is designed to be fully general in that every possible combinatorial bid is available to the bidders. If, however, the auctioneer wishes to restrict the bids in any manner that he finds convenient to verify, the auction structure will accommodate this, and the auctioneer can announce to the bidders a list of attributes a bid must have. (An example of such an attribute might be: “Bids that are combinatorial are to be composed of geographically contiguous subsets of the properties but within each property may include any subset of the spectrum bands.”) This is formalized below.

2.1 Definitions

Label *properties* $j \in J$, and *blocks* $k \in K$, where $K = K(J, A)$ is a set of subsets of J defined by a set of *attributes* A that are computationally tractable for the auctioneer to verify for each member of K. Let

$$K_n = \{k \in K(J, A) : 1 \leq |k| \leq n\},$$

where $|k|$ is the number of properties in block k. A *partition* $P = (p_1, p_2, \ldots, p_r)$ is a collection $p_1, p_2, \ldots, p_r \in K$ such that $\bigcup_{i=1}^{r} p_i = J$, and $p_i \cap p_j = \emptyset, i \neq j$. A *composite bid* comprises a partition $P = (p_1, p_2, \ldots, p_r)$, together with an *evaluation*:

$$(C(P); c(p_1), c(p_2), \ldots, c(p_r))$$

where

$$C(P) = \sum_{i=1}^{r} c(p_i), \tag{1}$$

and $c(p_i)$ is the *bid* for block p_i.

To be more precise, $c(p_i)$ is the *value of the bid for block* p_i. A composite bid consists of $3r + 1$ pieces of information, capable of registration in a database. The first piece of information is the total value of the composite bid, $C(P)$. The $3r$ pieces of information are, for each i ($i = 1, 2, \ldots, r$): (1) the specification of the block p_i, (2) the value of the bid on the block $c(p_i)$, and (3) the identity of the bidder for block p_i. All $3r + 1$ pieces of information are available from the database to all bidders.

2.2 Two Stages of the PAUSE Auction

Stage 1: Bidding on Individual Properties

The Bidders. Each bidder submits a collection of bids on individual properties. In each round there is an *improvement margin requirement*:

> The new bid must improve on the previous best bid on that property by *at least* ϵ and *strictly less than* 2ϵ.

The Auctioneer. In each round, for each property the auctioneer checks that a bid on that property is *valid* by checking:

> *Increment validity*: The bid satisfies the bounds of the improvement margin requirement.

In each round, the lowest valid bid on each property is accepted. The round ends when bidding ends on all properties. Stage 1 is divided into three substages. At the conclusion of the third substage, the leading (i.e., lowest) bids on the properties are registered to their respective owners.

Activity Rules. A bidder is designated *active* on a property if he has the leading bid from the previous round or submits an acceptable bid in the current round. Each of the three substages contains an unspecified number of bidding rounds. The bidders must remain active on properties covering, respectively in the three stages, 60 percent, 70 percent and 80 percent of the number of subscribers for which they wish to remain eligible to bid. The transition from substage 1 to 2 occurs when there are bids on no more than 10 percent of the subscribers for three consecutive rounds, from substage 2 to substage 3 when there are bids on no more than 5 percent of the subscribers for three consecutive rounds.

Stage 2: Combinatorial Bidding

The Bidders. Each bidder submits a single composite bid (which, by definition, includes all the properties in the auction). In each round there is an *improvement margin requirement*:

> Let b be the number of *new* bids in the composite bid. The new evaluation must improve on the previous best evaluation by *at least* $b\epsilon$ and *strictly less than* $2b\epsilon$ (i.e., an average improvement per block of at least ϵ but less than 2ϵ).

Each bidder's partition $P = (p_1, p_2, \ldots, p_r)$ is restricted to $p_i \in K_n$, where $c(p_i)$ is either a new bid for block i, or a registered bid. Initially, $n = 2$.

The Auctioneer. In each round, the auctioneer checks that a composite bid is *valid* by checking:

(i) *Bid validity*: Each bid which is asserted to be registered in the database is indeed so registered, and that new bids identify correctly the bidder for block p_i, and satisfy $p_i \in K_n$.

(ii) *Evaluation validity*: Equation (1) holds, i.e., the value $C(P)$ of the composite bid is indeed the sum of the bids on each of its blocks, and

(iii) *Increment validity*: The bid evaluation $C(P)$ satisfies the bounds of the improvement margin requirement.

In each round of Stage 2, the new collection of bids on the blocks $\{c(p_i)\}$ are registered in the database to their respective owners, and the lowest valid composite bid is accepted. Thus, in each round, the auctioneer accepts *one* composite bid from among all the composite bids submitted by the players,[5] but registers all the valid composite bids. A substage ends when bidding ends; n increases in the following substages.

[5]This is in contrast to the AUSM scheme.

The size of the bid increment, ϵ, and the rate of increase of the block size limit, n, are used by the auctioneer to control the speed of the auction, in conjunction with the activity rules. (Figure 1 provides a simple illustration of combinatorial bidding.)

Activity Rules. A bidder is *active* on a property if his bid on a block containing that property forms part of the accepted composite bid of the previous round, or if he submits a valid bid in the current round on a block containing that property. Each of the substages contains an unspecified number of bidding rounds. The bidders must remain active on properties covering, for example, 90% or 98% of the number of subscribers for which they wish to remain eligible to bid. The transition between substages occurs when there are bids on no more than 10 percent of the subscribers for three consecutive rounds.

It is essential that, before the start of Stage 2, the auctioneer specifies the rules that need to be satisfied by a valid composite bid in a manner that can be checked by players, as well as by the auctioneer.

2.3 Illustration of Combinatorial Bidding

To see how the auction procedure works, consider the small example consisting of 4 players, a, b, c, and d, bidding for 2 spectrum bands in each of 3 geographic areas, for a total of 6 properties. In the illustration, the spectrum bands are represented by the rows, and the geographic areas by the columns. For purposes of this illustration, will ignore bid increments and other non-combinatorial aspects of the auction procedure.

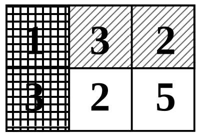
The first diagram shows the results of the final round of Stage 1, where the leading bids are shown; all the players except c has a leading bid on at least one property. The total value of the accepted bids is 16.

In round 1 of Stage 2, player b is leading on two blocks of size 2, and has increased the total bid on each of them from 5 to 6. He fills out the remainder of his composite bid with two earlier bids on the remaining properties from players a and d. From the diagram, can assume that the auctioneer chose player b's composite bid because its

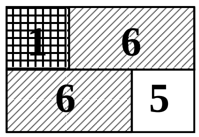
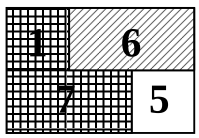
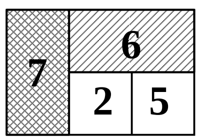
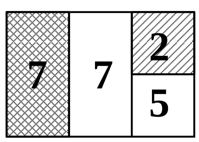
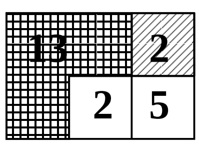
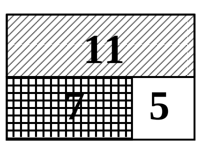
Numerical Illustration

Players: a b c d

STAGE 1

	Total value of accepted bids	Individually submitted by
	16	a, b, d

STAGE 2

		Evaluation of accepted composite bid	Submitted by
(*n* =2) Round 1		18	b
Round 2		19	a
Round 3		20	c
Round 4		21	d
(*n* =3) Round 5		22	a
Round 6		23	b

evaluation of 18 was higher than the evaluation of all other composite bids, improving on the previous round's value of 16.

In round 2 of Stage 2, player a has submitted the chosen composite bid, which has an evaluation of 19. Player a's composite bid is made up of the same set of bids as the previous round, with one change: he has improved on player b's bid on the first two properties in the lower row from 6 to 7.

Stage 2 continues in this way until Stage 5, at which point bidding is allowed on blocks of up to size $n = 3$. The auction continues one more round and concludes with a total evaluation of 23, when no more bids were submitted by any of the players.

2.4 Auction Rules

Bid Waivers. The auction can include bid waivers, especially if the time between rounds is short. In Stage 1, the number of waivers would be concurrent with the number of waivers issued in the PCS auctions, and in Stage 2 a number 1.5 times this amount.

Bid Withdrawals. No bid withdrawals are allowed in either stage. It may be asked why they were permitted in the PCS auction, since they complicate the auction. P. Milgrom (1996a), in his attachment to GTE's comments, explains: "In effect, a bid withdrawal substitutes partially and quite imperfectly for combinatorial bidding." Porter (1999) reports results of experiments on auctions with bid withdrawals with penalties. He found that efficiency and revenue increase, but individual losses are larger. He also found that the increased efficiency does not outweigh the higher prices paid; thus, the bidder surplus falls.

2.5 Auction Variations

Multiple Winners. A variation of this procedure that allows for multiple winners, such as may be desirable in an auction for Carrier of Last Resort responsibility, is presented in Kelly and Steinberg (2000). The design described there also incorporates the feature of minimizing against the opportunity for bidder collusion to which such an auction might otherwise be susceptible.

Overlapping Bids. For simplicity of exposition, we have assumed that the properties

$j \in J$ are disjoint, but other possibilities may be of interest. For example, in the auction of frequency bands for spectrum, there may be two or more incompatible band plans, with a simple definition of disjoint properties within each band plan. In this case, Stage 1 could be used to set an initial price for each property in each band plan, with a composite bid in Stage 2 relating to a player's preferred band plan.

Boolean Bids. A bid $c(p_i)$ is contingent in the sense that the bidder is offering to serve all the properties in the block p_i or none. More general forms of contingency could be allowed through Boolean bids. A Boolean bid is a string of blocks together with Boolean 'and', 'or' and bracket (left bracket or right bracket) connectors, together with a value or values. The format of the Boolean bid would need to be specified in such a manner that, given a set of properties, it would be an elementary computational task to evaluate the value of the bid for that set of properties (with the value being infinity if the set of properties is not consistent with the Boolean bid).

3. The Propositions

PAUSE clearly allows for all combinatorial bids. Desiderata item (a), transparency, is addressed by requiring bidders in Stage 2 to submit composite bids. The auctioneer's procedure described in Section 2.2 for evaluating the combinatorial bids clearly is computationally tractable, assuming that the associated information storage problem is manageable. This is easily settled by Proposition 1, which addresses desiderata item (b). However, Proposition 1 first addresses desiderata item (c) by showing that the completion time of the auction is bounded. Propositions 2 responds to desiderata item (d), prevention of jump bidding and mitigation of the threshold problem.

PROPOSITION 1. BOUNDS ON NUMBER OF ROUNDS. *Since in each round of Stage 2 the value of the accepted composite bid must increase by at least ϵ over the previously accepted composite bid, the number of rounds in total is bounded above by $C_0(P_0)/\epsilon$, where $C_0(P_0)$ is the value of the opening composite bid.*

Let B be the number of bidders. Since each bidder is allowed to make at most one *composite bid per round, the maximum number of bids that needs to be registered by the auctioneer is bounded above by* $(C_0(P_0)/\epsilon)B|J|$.

Remarks. Although for the procedure we present here, the auctioneer's problem of determining the winning bid is computationally tractable, for a bidder it may be an NP-complete problem to determine whether he can make a composite bid that beats the currently accepted composite bid. However, there is very little computational burden for small players interested in only a small number of properties. If no synergies are claimed, then the auction reduces to an auction of the type utilized for the PCS licenses, viz., Stage 1 only. As discussed in the Introduction, the results of Rothkopf, Pekeč and Harstad (1998) show that, if the form of composite bids is restricted in one or other of several possible ways, then the problem can be made computationally tractable. However, in cases where the bidders are unlikely to agree upon the form of the appropriate restriction on composite bids, we view the elicitation of the form and size of potential synergies as a major purpose of the auction proposed here.

Work on computationally difficult problems shows that in several situations where finding the exact optimum is hard, finding a good approximation to the optimum with high probability may be relatively easy (Jerrum and Sinclair 1996). It is our belief that the traditional problems of elicitation and gaming are more serious difficulties than the possible computational burden on those bidders claiming complex synergies.

Proposition 2. Prevention against Jump Bidding and Mitigation of the Threshold Problem. *The rule that the improvement margin is bounded above by* 2ϵ *reduces the possibility of price-jump bidding. The progression of allowable block size* n *reduces the possibility of block-jump bidding and mitigates the threshold problem.*

Remarks. The upper bound on the improvement margin also tends to reduce the bidders' computation requirements, by limiting the range of possibilities that need to be considered. The existence of the block size also aids the auctioneer in controlling the speed at which the auction progresses.

Note that a price for a property cannot be determined from a composite bid if, within that composite bid, the property is part of a larger block.

4. Suggestions for Future Work

Like the PCS and AUSM auction structures, PAUSE is probably too complex to admit much theoretical analysis. Future work could focus on the determination of appropriate values for the auction parameters:

- *Eligibility Requirements:* Values of the eligibility parameters A_i, that determine that a bidder is designated as active.

- *Bid Increments and Block Size:* Values of ϵ, and the progression of n in Stage 2. For example, the auctioneer might move n from the starting value of 2, to 3, 4, 5, ...; however, the auctioneer might instead move n to 4, 8, 16,..., if he would like to increase the pace of the auction and sees no signs of a free-rider problem. In either case the value of the bid increment ϵ would decrease, and the activity rule percentage A_i increase, as n increases.

- *Transition between Substages:* Percentage of the population base, T_i for all i, that determines the transition to the next substage. (The FCC has found that, on occasion, it has been desirable to make *ad hoc* modifications to the T_i values.)

Future work could also consider other auction rules:

- *Bid Waivers:* As pointed out in Section 2.4, the auction can include bid waivers. We suggested that the number of bid waivers in Stage 1 correspond to the number of bid waivers in the PCS auctions, and in Stage 2 a number 1.5 times that amount. However, the appropriate number of bid waivers needs to be determined.

- *Bid Withdrawals:* As explained in Section 2.4, bid withdrawals can be viewed as an imperfect substitute for combinatorial bidding, and thus in this auction there would probably be no need for bid withdrawals. However, testing of this hypothesis would probably be of value.

In this paper, we have focused on the computational challenges involved in combinatorial auctions, an essential step before testing. With the results of testing, it will be possible to explore the allocative efficiency of auction designs.

5. Discussion

At and immediately following the May 2000 Combinatorial Bidding Conference at Wye River, Maryland, several issues were raised. It was suggested that the PAUSE auction would not handle the situation when players have *only* synergies–i.e., do not value any individual properties–since in such a circumstance, the auction would not run at all as Stage I will not be operational. However, the PAUSE auction is designed as an enhancement to the SMR auction, i.e., to capture additional surplus due to synergies; for the circumstance just described, neither would the SMR auction run. (Such a situation may indicate that the auction was poorly designed, e.g., the properties were chosen to be too small.)

Another suggestion was that the auction design requires budget constraint to insure that a composite bid makes use of bids from a player in a single round. However, a player could always choose to include a budget constraint through the use of a Boolean bid (e.g., bid a or bid b but not both).

Two other issues are the efficiency of this procedure and its susceptibility to strategic gaming. We examine these issues through a series of simple examples. Consider two players, A and B, bidding for two properties X and Y, where package bids on the combination $\{\mathrm{X}, \mathrm{Y}\}$ are permitted.

In the first example (Table 1a), the efficient outcome is for A to be allocated Y and B to be allocated X, for a total surplus of 11. The bidding is shown in Table 1b, where B earns a surplus of 4 on property X, and has no interest in bidding 6 for the package in Stage 2, which would result in a surplus of only 3 for him. Similarly, A earns a surplus of 2 on property Y, and has no interest on the package in round 4. The outcome is efficient.

Now consider the example of Table 2a, where the efficient outcome is for B to win both properties. The bidding is shown in Table 2b. The auction ends in the first round of Stage 2, because A has no incentive to make *either* a package bid (since the value to A of XY is 6, and so A has *no* surplus for a bid of 6, and the bid would need to be 7) *or* to increase his bid on Y (since an increase to 5 would be necessary, but the value to A is only 4). The outcome is efficient.

The third example is given in Table 3a. Table 3b shows the result when both players bid in a straight-forward fashion. In the first round of Stage 2, B makes a package bid of 6, which gives him a surplus of 5. In the second round of Stage 2, Player A makes a composite bid of 7 (combining B's bid of 2 on X with a new bid of 5 on Y), giving A a surplus of 1. The outcome is efficient.

What happens if bidders act strategically? Specifically, suppose that B does *not* bid in round 2 for property X. (See Table 3c). The end result is that player B wins both properties with a package bid of 7. Here B "knew" that if bid 2 for X in round 2 of Stage 1, it might be used against him later. This possibility for strategic bidding, which did not exist in the second example, results in an inefficient outcome.

Tables 4a and 4b presents an example due to David Lucking-Reiley. Here, the outcome is inefficient, but there was no strategic bidding, and the revenue generated is greater than would have been generated by the efficient outcome.

These simple examples show that the PAUSE mechanism may not be efficient. However, the advantage of allowing for synergies may overshadow the possible inefficiency of the auction.[6]

[6]This paper is based on a companion paper, Kelly and Steinberg (2000), a preliminary version of which was submitted *ex parte* to the FCC by the Citizens for a Sound Economy Foundation (Kelly and Steinberg 1997). I am most grateful to Frank Kelly for discussion and suggestions. In addition, I would like to thank the participants at the May 2000 Combinatorial Bidding Conference at Wye River, Maryland, with special thanks for specific comments from Jeremy Bulow, Ian Gale, David Lucking-Reiley, Paul Milgrom, and Charlie Plott.

	X	**Y**	**XY**
A	1	**5**	7
B	**6**	2	9

Table 1a: Example 1.
Efficient outcome indicated in bold.

	X	**Y**	**XY**
Stage 1			
1	A = 1	A = 1	
2	**B = 2**	B = 2	
3		**A = 3**	
Stage 2 ($n=2$)			
No bidding			

Table 1b: No package bidding occurs.
Outcome is efficient.

	X	**Y**	**XY**
A	1	4	6
B	6	2	**11**

Table 2a: Example 2.
Efficient outcome is for Player B to win block XY.

	X	**Y**	**XY**
Stage 1			
1	A = 1	A = 1	
2	B = 2	B = 2	
3		A = 3	
Stage 2 ($n=2$)			
1			**B = 6**

Table 2b: Package bidding.
Outcome is efficient.

	X	**Y**	**XY**
A	1	**6**	7
B	**6**	2	11

Table 3a: Example 3.
Efficient outcome indicated in bold.

	X	**Y**	**XY**
Stage 1			
1	A=1	A=1	
2	B=2	B=2	
3		A=3	
Stage 2 ($n=2$)			
1			B=6
2	[**B=2**]	**A=5**	

Table 3b: Straight-forward bidding.
Outcome is efficient.

	X	**Y**	**XY**
Stage 1			
1	A=1	A=1	
2		B=2	
3		A=3	
Stage 2 ($n=2$)			
1			B=5
2			A=6
3			**B=7**

Table 3c: Strategic Bidding.
Outcome is inefficient.

	X	**Y**	**XY**
A	1	**9**	13
B	**8**	4	15

Table 4a: Example 4.
Efficient outcome indicated in bold.

	X	**Y**	**XY**
Stage 1			
1	A = 1	B = 1	
2	B = 2	A = 2	
3		B = 3	
4		A = 4	
Stage 2 ($n=2$)			
1			B = 7
2			A = 8
3			B = 9
4			A = 10
5			B = 11
6			A = 12
7			**B = 13**

Table 4b. Straight-forward bidding.
Outcome is inefficient.

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