Tradeoffs in Combinatorial Auction Design

Implications for the FCC spectrum auctions

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Overview

- Disclaimer: auction design is not a secondary issue
- Desirable properties of (combinatorial) auction models
- Difficulties in multiround combinatorial auction design
- Tradeoffs:
  - Computational issues
  - Any hope for ascending combinatorial auctions?
  - Coping with inherent cooperative nature
- Some comments on two-sided combinatorial auctions
- Summary of proposals

Disclaimer

Auction design is not a secondary issue

- **Overall 3G strategy**

  - issues of standardization

  - simultaneous or sequential sales

- **Selling licenses or selling rights to be blackmailed?**

  - UHF broadcasters

  - carte blanche or designated use of frequencies

Required reading ☺: *Some Heretical Thoughts on the Design of Combinatorial Auctions for the FCC* by M.H. Rothkopf (see conference web site)

Disclaimer

Working assumptions

- analysis of design and modeling issues:

  not focused on economic theory of auctions

  not focused on CS/OR theory

- focused on specific application: the FCC auction design problem

- some issues might be irrelevant in different contexts

  (e.g., B2B like procurement, repeated sales, small stakes, …)

- as in any model analysis, bits and pieces can’t be taken out from different models and patched together to one’s liking.

Running Example

FCC's upcoming 700MHZ auction (June 2002?)

700 MHz Band EAGs

	Northeast	Mid-Atlantic	Southeast	Great Lakes	Central / Mountain	Pacific
10 MHz	WXEAG701-C	WXEAG702-C	WXEAG703-C	WXEAG704-C	WXEAG705-C	WXEAG706-C
20 MHz	WXEAG701-D	WXEAG702-D	WXEAG703-D	WXEAG704-D	WXEAG705-D	WXEAG706-D

Combinatorial Auction

Set of items to be sold: [n] = {1,2,…,n}

"All or nothing" bids allowed for any combination S $\subseteq$ [n]

Each item can be sold to at most one bidder.

Winner determination problem (WDP)

If the goal is to maximize the total revenue, then WDP is equivalent to weighted set packing on hypergraphs.

(Rothkopf et al. 95,98)

- Interesting algorithmic issues. Does standard TCS approach help?
- Any good auction theory for combinational auctions?

Why Comb. Auctions?

- Gives more expressive power to bidders
- Inherent complexity plagues design and implementation

How much and what type of trade-off?

Why not prepackage and avoid complexity issues?

Why not deal with complexity (heuristics, approximation,…)?

Why not limit behavior by imposing procedural rules?

Application specific design issues

Desirable Properties?

- Fairness
- Failure-freeness
- Allocation Efficiency
- Revenue Optimization
- Low Transaction Costs
- Transparency
- Scalability
- ....

Fairness

Settling for a suboptimal allocation

- Heuristics
- Relegating complexity to the bidders (e.g., AUSM, PAUSE)
- “Political” solutions
- …

Allocate items to those who value them the most.

OR

… to those who are the luckiest

… to those who compute the best

… to those who complicate / manipulate the auction procedure the most

Failure-freeness

Heuristics (IP attacks; AI)

Approximation algorithms

...

Work most of the time

- Fine in small stakes auctions. Could be fine in procurement.

➢ How about missing the optimal allocation for a $30 billion auction?

➢ How about miscalculating current winners in only one round of a multiround auction?

Auction designer’s dilemma

Are you ready to accept the following gamble:

$p_{failure}$

Revenue within (1-ε) of optimal
“nearly” optimal allocation

$1-p_{failure}$

Revenue maximized
Optimal allocation

<u>Depends on</u>

ε

$p_{failure}$

alternatives

Auction designer's dilemma

Are you ready to accept the following gamble:

$p_{failure}$

- **Lawsuit, long delay in allocation**
- **Ramifications for the industry, consumers, taxpayers**
- **Public embarrassment**

$1\text{-}p_{failure}$

Revenue maximized
Optimal allocation

<u>Depends on</u>

$p_{failure}$

alternatives

Auction designer's dilemma

Estimating $p_{failure}$:

- experimental data? benchmark instances?

- modeling bidder behavior with probability distributions

➢ beware of malicious bidders (could enter auction as such or could change its goal during the course of the auction).

➢ multiple rounds: multiple possibilities of failure

Rev. Max. vs. Efficiency

- **Theoretical results vs. implementability in real life**
- **The most important segment:**

  citizens?

  taxpayers?

  consumers?

  industry?

VCG mechanisms

Bidders have to have valuations for all possible allocations.

Allocation?

a) Partition of sets of winning combinations that belong to the same bidder

or

b) Partition of winning combinations (two or more can belong to the same bidder)

or …

VCG mechanism is context dependent

VCG mechanisms

- Find winning combinations/bidders by solving WDP (maximizing revenue)
- For each of the winners find the lowest amount that this bidder could bid and still be a winner (keeping all other bids unchanged). This is the price paid by that bidder.

no incentive for strategic bids

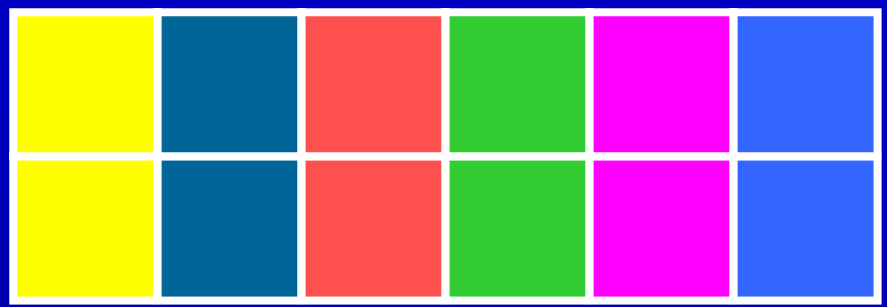
outcome is efficient

However:

- McCabe et al. (1991), *Testing Vickrey's and other simultaneous multiple unit versions of the English auction*

- Hobbs et al (2000), *Evaluation of a Truthful Revelation Auction for Energy Markets with Nonconcave Benefits*

VCG Example



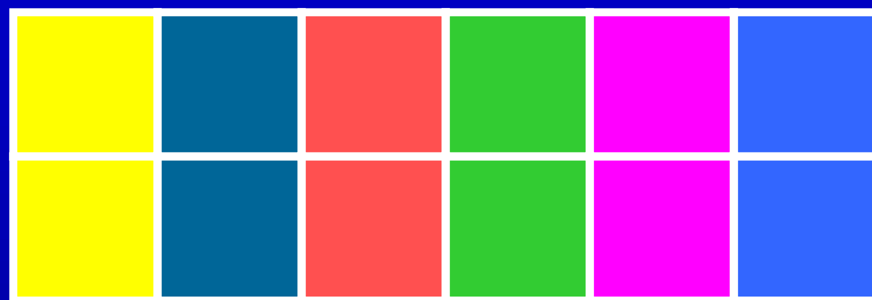
Suppose seven bids (in $billion) only:

$b_1(\;)=4$, $b_2(\;) = b_3(\;) = b_4(\;) = b_5(\;) = b_6(\;)= b_7(\;)= 1$

- bids 2,3,4,5,6,7 define the revenue maximizing collection
- EACH WINNER PAYS NOTHING!

• Should bid-taker aim for an efficient outcome?
• Should bid-taker care about VCG mechanisms?

VCG Example 2

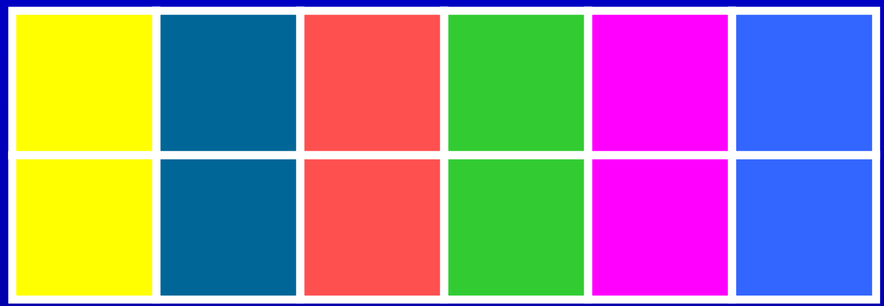


Suppose

Alice bids $b_1($$)=4$

Bob wants to bid $b_2($$)=6$

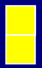
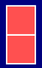
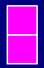
VCG Example 2



Suppose

Alice bids $b_1($$)=4$

How about the following bids for Bob:

$b_2($$) = b_3($$) = b_4($$) = b_5($$) = b_6($$)= b_7($$)= 1$

Transparency

Trust issue: The auction rules and procedures ought to be transparent to all bidders

Complexity vs. Transparency

WDP, min bid increment, tie-breaking,…

Should we assume poly-time $\Leftrightarrow$ transparent ?

- **Transparency and bounded rationality**
- **Programmed bounded rationality (AI)?**

Scalability

Important in repetitive use of the auction procedure

For example:

high complexity procedures like B&B based IP solvers

- do the job for small n (single-round vs. multiround matters)
- there are no guarantees for large(r) n.

Multiround?

In multiround framework problems escalate:

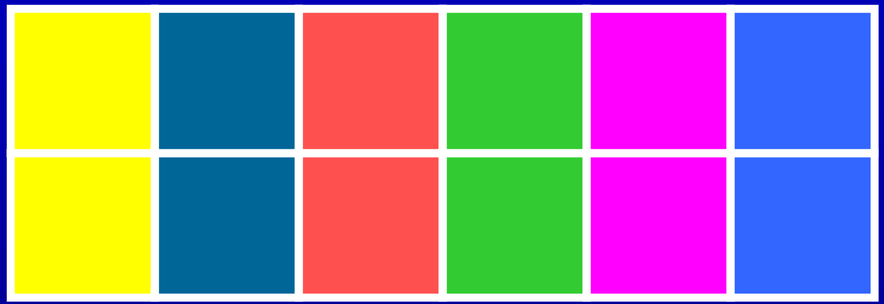
WDP, threshold problem, exposure problem

The source of many problems: currently losing bid can become winner later due to action on other items.

- invites bidder coordination and collusion
- complicates the selection of active bids

Threshold Problem

the difficulty that multiple bidders desiring combinations that constitute a larger one may have in outbidding a single bidder bidding for that larger combination



Suppose seven bids (in $billion) only:

$b_1(\;)=7$, $b_2(\;) = b_3(\;) = b_4(\;) = b_5(\;) = b_6(\;)= b_7(\;)= 1$

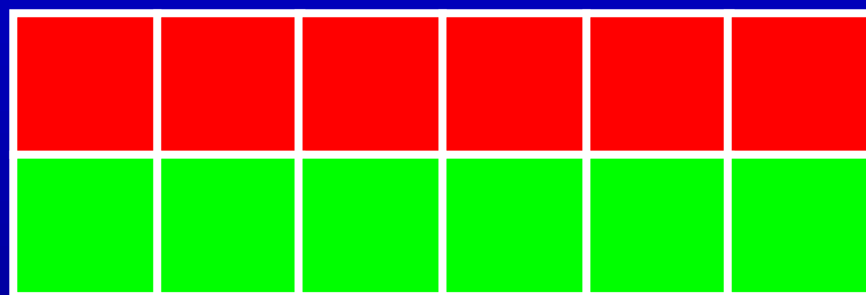
Any of the bidders 2-7, if acting alone, has to double their bid.

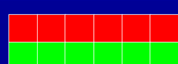
Allow coordination?
Allow “deficiency sharing”?

Exposure Problem

the risk of bidders winning items they do not desire

(More serious problem in simultaneous single item auctions)



Suppose b() = 6

Suppose Alice wants to bid b()=5 XOR b()=3

(does not want both  and )

Allow for contingencies (XOR bids, budget constraints,…)?

Problem in multiround format

Challenges in Multiround

- Procedures for prevention of strategic bidding and signaling
- Procedures for keeping bidding moving
- Procedures that mitigate the exposure problem
- Minimum bid increase determination procedures
- Tie-breaking procedures

FCC Design: Version 5/00

- Limit biddable combinations to individual, regional, national, global licenses (as in Rothkopf et al. 95,98)
- From round to round retain only high bids on each biddable combination
- Min bid in the next round: x% higher than the current high bid on that combination.
- Auction ends after two rounds without new valid bids

Limited expressive power

Exposure problem?

Slow pace?

FCC Design: Version 7/00

- Limit the number of bids on combinations (12/bidder)
- Only current wins retained from round to round

  (renewal also possible)
- Min bid in the next round: max of

  - x% higher than the bidder's previous high bid on that combination (can still be well below current high bid)
  - #units * min per unit price of any current winner in last 5 rounds
- WDP: current winners but

  - only bids from each bidder's last two active rounds
  - bids across rounds are mutually exclusive for each bidder

FCC Design: Version 7/00

- **overconcerned with the exposure problem?**
- **what if only single item bids submitted?**
- **endless gaming possibilities? (eligibility, activity credit)**
- **slow pace?**
- **scalability?**
- **transparency?**
- **tie-breaking rules?**
- **dubious interpretations of algorithmic and complexity issues?**

Required reading ☺: *The FCC Rules For the 700MHz Auction: a Potential Disaster by* C. R. Plott

(http://www.fcc.gov/wtb/auctions/31/releases/rules6.pdf)

Tradeoff

Allowing bids on all packages

vs.

Failure-freeness, Fairness, Scalability

How theoretical computer scientist's worst nightmare
enters (without an invitation) the auction designer's life

Computational issues

Uneasy facts of life:

- **WDP is NP-complete**

- WDP is **as hard as NP-complete problems get** (somewhat irrelevant since any non-optimal allocation, whether the corresponding revenue is ε–close or far away from the optimal revenue, smells like a disaster in the FCC case)

- Can solve instances that are not too "large". However, if required to solve multiple instances (no matter how similar) **do not count on "economies of scale"**

- **Beware of savvy salesmen** ("great heuristics" that work well on self-selected database of test problems) since track record on hard IPs (e.g., TSP, QAP) shows that *branch-n-cut* is the way to go and that one could take "forever" (from FCCs perspective).

Who Cares?

Many of these issues/problems can be brushed off:

1. Accept possibility of suboptimal WDP solutions
2. Provide only minimal feedback to the bidders:
   - current winner (Yes or No)
   - minimal bid increment in order to remain active (calculated to your liking)

Easy way that avoids some potential pitfalls. Is it the best way?

More importantly: is FCC in position to do any of these?

Coping with Complexity

- IP approaches (e.g., B&B. B&C)
- Heuristics: approx. alg’s, AI approach,…
- Relegating complexity to the bidders (AUSM, PAUSE,…)
- Maintaining fairness in face of computational limits
- Limiting biddable combinations
- Limiting use of combinatorial bids

Coping with Complexity

- IP approaches (e.g., B&B. B&C) scalability? transparency?
- Heuristics: approx. alg's, AI approach,… failure-freeness?
- Relegating complexity to the bidders (AUSM, PAUSE,…) fairness?
- Maintaining fairness in face of computational limits fairness?
- Limiting biddable combinations fairness?
- Limiting use of combinatorial bids fairness? failure-freeness?

Coping with Complexity

- **Relegating complexity:**
  - allocating to those who compute the best
  - dealing with complexity should not be FCC’s task or responsibility

- **“Political” solutions:**
  - an elegant way out (is it good enough to deter lawsuits?)

- **Limiting biddable combinations:**
  - cannot be avoided (e.g., frequency blocks, regional blocks)
  - a matter of framing the issue?
  - possibly the cheapest way out

    (since you have to bite the bullet somewhere)

Outsource Computation

Combinatorial Auctions $\Rightarrow$ WDP $\Rightarrow$ Computational Issue

Computational problem should not plague allocation decisions

WDP auction

- Don’t have to be comb. auction bidder to participate
- Submissions are allocations in comb. auction
- Winner: bidder submitting an allocation with highest revenue
  (one who computes WDP the best)
- Payout: flat fee, percentage of the revenue value, Vickrey-like,…???
- Mechanism: ???

Instead of being courted by WDP computation experts, let the market decide.

Tradeoff

Ascending Combinatorial Auctions:

Theory vs. Reality

Free lunch – a myth or reality?

Ascending Comb. Auctions

Some serious complaints (not discussed here):

- allow gaming and bidder collusion
- slow pace
- redundant when combinatorial bidding allowed

Required reading ☺: *Some Heretical Thoughts on the Design of Combinatorial Auctions for the FCC* by M.H. Rothkopf (see conference web site)

2^{nd} Price Comb. Auctions

Winners:
bidders/combinations defining revenue maximizing collection

Revenue:
no less than the value of the revenue maximizing collection when only all non-winning bids (bidders) considered

Problem: distributing "surplus" (FP-SP) among winners

Let $P(w_{i1},\ldots,w_{ik})$ = max revenue if bids from winners $w_{i1},\ldots w_{ik}$ removed.

For any collection *C* of winners, let **d(*C*)=FP-P(*C*)**

Distributing "surplus" ⇔ a solution to cooperative game **d**

Note: core of d is empty (except in the trivial case)

IP duality gap hits hard

Required reading ☺: *LP and Vickrey auctions* by Bikhchandani et al.
http://www.anderson.ucla.edu/faculty/sushil.bikhchandani/papers/vick_lp.pdf

➢ explores existence of primal-dual algorithms to construct ascending auctions that implement Vickrey outcome.

➢ crucial property: “agents are substitutes”
“If the substitutes condition does not hold, we believe it is unlikely that an iterative auction (in which bidding sincerely is an equilibrium) yielding Vickrey outcome exists.”

BAD NEWS for comb. auctions (straightforward to check)

“agents are substitutes” holds ⇔ Core of d is not empty

➢ Back to the complexity issue: WDP is an NP hard IP problem. Duality gap exists and this means that the machinery of primal-dual algorithms is not available.

Ascending comb. auctions

- Efficient outcome might require simplifying assumptions

  - E.g., IPV assumption might help but should FCC care?

Efficient ascending combinatorial spectrum auctions:
a myth or reality?

- Plenty of issues that have to be juggled (on top of WDP)
  - min bid increment/ eligibility / activity credit
  - exposure problem / threshold problem

Simple rules in ascending combinatorial spectrum auctions:
a myth or reality?

(good news for would be consultants)

Tradeoff

Inherent Cooperative Nature of Combinatorial Auctions:

Mitigating exposure problem and threshold problem

vs.

Avoiding bidder collusion, signaling, and eligibility parking

Procedural issues

- **Eligibility/activity credit**: is extending SAA rules the only way?
- **Minimum bid increment:**

  - a notion of “fair share”of “*shortfall*”

  - has to ease threshold problem

  - has to ease the exposure problem (does it really?)

  - has to minimize gaming possibilities

Contingencies

XOR bids: $\mathbf{b(C_1)}$ XOR $\mathbf{b(C_2)}$ XOR … XOR $\mathbf{b(C_k)}$

- how much is IP messed up with addition of XOR bids?

- introduce dummy items

- works w/o trouble for most relevant cases where WDP is tractable

(e.g., FCC v.5/00, but have to be careful)

Budget constraints: $\text{XOR}\{ \underset{i \in I}{\text{OR}}\; b(C_i) : \sum_{i \in I} b(C_i) \leq M \}$

- knapsack problem

Procedural constraints (e.g., FCC v.7/00)

Min bid increment

W – winning (revenue maximizing) allocation

For any biddable combination C, calculate

Gap(C)=Rev(*W*)-max{Rev(*A*): allocation *A* contains C}

- Algorithm for WDP can (and often has to) be used for calculating Gap (Rothkopf et al. 95,98)

- If min bid increment is based on Gap(C), auctioneer should make sure that these calculations are (as) failure-free (as possible) and doable in time provided between the rounds.

(FCC v.7/00 has to be ready for potentially 4095 such calculations)

Min bid increment

Two extremes:

- **Immediate impact:**

  Set min bid increment for C at mbi(C) = Gap(C)

- **No impact:**

  Set min bid increment for C at mbi(C) = (x%)* b(C)

Neither of the rules relies on possible increases on complementary combinations that could help bridge the gap.

Min bid increment

How about bridging only a "fair share" of the gap?

$$\mathbf{mbi(C) = m_C(C) * Gap(C)}$$

where m_C is a measure, possibly different one for different C, on the algebra of all combinations S that are disjoint from C or that contain C.

Examples:

$m_C(S)=|S|$

$m_C(S)=m(S)$ for some m that is fixed prior to auction

$m_C(S)= b(S)/ \max\{Rev(A): A \text{ contains } S\}$

Appropriate m?

(mbi proposed by DeMartini et al. 1999

and Vohra and Weber 2000 are special cases)

Min bid increment

The problem with mbi(C) = m_C(C) * Gap(C)

- implicitly assumes that Gap(S)=Gap(C) for all combinations S in allocation *A** = arg max {rev(*A*): *A* contains C}

For example, there can be S contained in both *A** and *W*.

C=this row:	**2**	**2**	**2**	**2**	**2**	**2**	10 = b(C)
S=this row:	1	1	1	1	1	1	**8** = b(S)

Rev(***W***)=20

For such S, 0=Gap(S)<Gap(C). Thus, mbi(S)=0. So, even if all other bids were increased by a “fair share” , mbi(C) would not be sufficient to bridge Gap(C).

Min bid increment

$$mbi(C) = m_C(C) * Gap(C)$$

Should a "fair share" be computed more precisely?
Should new bids stand a chance to be immediately competitive?

A better approximation:

set constraint on m_C: S in some $W \Rightarrow m_C(S) = 0$

More general:

Find $mbi(C) \geq 0$ such that for every C

$$\max\{ Rev(A) + \sum_{S \text{ in } A} mbi(S) \ : \ A \text{ contains } C \} = Rev(W)$$

Can do it by brute force. Tractable if WDP is tractable.

Before releasing, update mbi(C) by adding, e.g., (x%)*Rev(*W*) to it.

Min bid increment

- If no consensus measure m_C , then eligibility parking spots for some bidders (it seems that common value assumption is needed)

- Incorporating temporal information?
 - if no activity for some number of rounds on combination C, discount probability that there will be activity in the next round.
 (could easily be incorporated in the proposed framework)

Ties in comb. auctions

6	**7**	**6**	7	**6**	5	**40**
2	**3**	**4**	2	**4**	4	**20**
10	9	**10**	**10**	**10**	**10**	**60**

- More complex possibilities for ties
- Ties more likely with bidding agents (e.g. clickbox)

unless wise choice of opening bids and/or increments:

$\Rightarrow$ orders partitions

(determines tie-breaking up to the partition)

Should some partitions be favored?

Tie-breaking

Labeling and Aggregation

- Label each bid (e.g., time-stamp)
- Lexicographically order labels of all bids in an allocation
- Among tied allocations, find the extremal one in lex order. (e.g., allocation whose all bids were submitted first wins).

Alternatives available: (weighted) average label

Some bad ideas: uBid.com

Tie-breaking

Randomization

Select an allocation (uniformly) at random from the set of tied ones.

or

In clickbox bidding: perturb available bid increments by adding, e.g., $c(.5)^B$ (c is a constant, B are different and selected at random for each biddable combination)

Some bad ideas: FCC v.7/00

(randomize input for WDP algorithm)

Two-sided spectrum auctions

- asymmetry of participants (FCC and everyone else)
- what is being auctioned off (frequencies or frequencies for specific use)
- bundling rules (who, how and when)
- initial offerings vs. resale

STEP 0: Initial offering

Still one-sided auction

Note:

- Allowing broadcasters to sell their UHF rights to telecoms for 3G use is equivalent to allocating 3G spectrum to broadcasters

  (might make some people very happy at the expense of taxpayers)

- Introducing two sided-auctions: have to think hard of implications to current spectrum holders.

Playing with fire

If two-sided spectrum auctions are introduced:

Will FCC be able to control what's going on?

- beware of market manipulators
- beware of collusion of big players
- designating exchange facilitator, even if FCC decides to do this itself, might contribute to the market failure: the biggest players will eventually take over the market (numerous examples from B2B world)

Two-sided comb. auctions

The problem of dividing the winning package bid among multiple sellers:

- mandate using fixed underlying measure defined on all combinations? (e.g. MhzPops)

  Opens gaming possibilities (reminiscent of min bid increment analysis)

- risky: measure based on non-winning bids

  E.g., Bid(AB)=6, Bid(A)=1 Bid(B)=3

  Seller of A gets 6* 1/(1+3) = 1.5, Seller of B gets 6*3/(1+3)=4.5

  Problems similar to dividing "surplus" in 2^{nd} price comb. auction

  Possibilities for gaming: Seller of A could submit Bid(A) = 2.99

Two-sided comb. auctions

A possibility:

- FCC should ensure efficiency of initial offerings
- FCC should oversee transactions on secondary markets

  - design rules and regulations

  - allow only transactions that don’t change the designated use of frequencies

(alternatively, sales of frequencies without designated use; should not be retroactive)

Summary

At least three models better than FCC design v. 7/00:

- FCC v. 5/00 after fixing (e.g., according to P. & Rothkopf comments)

- SMR

- 1st price sealed-bid combinatorial auction that outsources WDP computation.

Summary

Define Policy Guidelines:

- importance of: citizens, consumers, industry, taxpayers…

- importance of various (un)desirable properties of allocation

- need for proper definition of failure and proper assessment of the probability and implications of a possible failure

Summary

Auction Design guidelines:

- Abandon FCC design v. 7/00 (or at least open discussion about it)

- rethink the need for multiple round auctions

  -clearly redundant if secondary markets will be created

- avoid taking responsibility for resolving NP-hard computation problems

  - limit biddable combinations

  - outsource computation

Do not forget

- bid withdrawals
- opening bids
- pace between rounds (if multiround)
- stopping rules (if multiround)
- defaulting (beware of malicious bidders)

…

Finally: Cannot study rules in isolation of other rules and the auction context

The devil is in the details.

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Implications for the FCC spectrum auctions

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Joint work with Michael H. Rothkopf, Rutgers University