The Power and Limitations of Item Price Combinatorial Auctions

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Talk Structure

- Types of Iterative Auctions
- Computational Power of Item-Price Auctions
- Complexity: item-price vs. Bundle-price auctions
- Some approximations by item-price auctions

Combinatorial Auctions

- *m* indivisible non-identical items for sale
- *n* bidders compete for subsets of these items
- Each bidder *i* has a valuation for each set of items: $v_i(S)$ = value that *i* assigns to acquiring the set *S*
  - v_i is non-decreasing (“free disposal”)
  - $v_i(\emptyset) = 0$
- **Objective:** Find a partition $(S_1 \ldots S_n)$ of $\{1..m\}$ that maximizes the social welfare: $\sum_i v_i(S_i)$
- **Issues:** communication, allocation, strategies

Iterative Auction Mechanisms

- At each stage, the mechanism presents to the next bidder i a set of tentative bundle prices $\{p_{i,S}\}$.
- The bidder at this stage ***should*** bid for the set S that maximizes his utility at these prices: $v_i(S) - p_{i,S}$.
- The mechanism rules determine: the prices at each stage, who bids at each stage, when to stop. Upon termination it should determine the final allocation and the payments.
- Types of natural restrictions:
  - **Item-prices** (linear prices): $p_{i,S} = \sum_{j \in S} p_{i,j}$
  - **Anonymous prices**: $p_{i,S} = p_{i',S}$
  - **Ascending Auctions**: $p_{i,S}$ non-decreasing with time

Why “should” bidders follow protocol?

- Many possible answers, with varying degrees of plausibility:
  - Incentive Compatible (in dominant strategies)
  - Charge VCG prices – ex-post-Nash incentive compatibility
  - Proxies – whether actual or cryptographically simulated
  - Myopic – this is exactly why these bids are intuitive
  - Obedient – A purely computational perspective
- In this talk we stay agnostic about incentives
  - Present whatever incentive properties obtained in each case
  - Impossibility results apply regardless of incentives

Can the efficient outcome be obtained?

- Parkes-Ungar and Ausubel-Milgron mechanisms reach the efficient allocation. They are ascending, non-anonymous, and use (non-linear) bundle-prices.
- **Open**: are there ascending *anonymous* mechanisms that reach the efficient allocation?
- **Our question**: are there *item-price* auctions that reach the efficient allocation?
  - If valuations are substitutes → yes — Kelso&Crawford
  - What about the general case?
  - Walrasian equilibrium with item prices does not exist.

Demand oracle view of item-price auctions

Demand Oracle Query:

- Input: m item prices: $p_1 \ldots p_m$
- Output: $D(p_1 \ldots p_m)$ -- Demand at these prices. I.e. the set S that maximizes $v(S)-\Sigma_{j \in S}\, p_j$

- A general item-price iterative auction may be viewed as an allocation algorithm whose input is a demand oracle for each bidder.

Lemma: A demand oracle can simulate a valuation oracle. (And, the simulation is computationally efficient.)

Valuation Oracle Query:

- Input: subset S
- Output: $v(S)$

Corollary: (Weird) item-price auctions can reach the efficient outcome and produce VCG prices.

Simulating a Valuation Oracle

Algorithm for computing $v(S)$ using a demand oracle:

marginal-valuation(j,S):

For all goods $j \in S$ set $p_j=0$; for all other goods set $p_j=\infty$

Perform a binary search on p_j to find lowest value with $D(p_1 \ldots p_m)=S$

v(S):

Initialize: result=0

For all goods $j \in S$ do

result ← result + marginal-valuation(j, $\{j' \in S \mid j'<j\}$)

Ascending Item-price auctions? No.

	v(a)	v(b)	v(ab)
Player 1	$\in (0,1)$	$\in (0,1)$	2
Player 2	2	2	2

- Finding an efficient allocation requires answering: $v_1(a)$ <> $v_1(b)$?
- Assume wlog that p_{1b} rose to 1 before p_{1a} rose to 1.
- Until this time, no information was gained (answer always {ab}).
- From this time on, no information on $v_1(b)$ can be gained.

Analyzing complexity of auction mechanisms

- Consider only informational costs; ignore computation
  - "preference elicitation", "communication complexity"
- Basic lower bound: every combinatorial auction requires exponential communication in worst case Nisan&Segal
- How to compare mechanisms?
  - How well they perform in real applications
    - We don't know. Not enough data. Life is hard.
  - How well they theoretically perform on classes of valuations
    - Semantic classes: "substitutes", "sub-modular", …
    - Syntactic classes: XORs of few bundles, ORs of few bundles, …
- What can we measure:
  - How much information transfer is needed to find socially efficient outcome.
  - How close to optimal can we get using "reasonable" information transfer.

Item-price vs. Bundle price auctions

- Bundle-price auctions can be exponentially faster then item-price auctions.
  - Assume all valuations are XOR bids of at most *s* bundles
  - No bidder will ever bid another bundle
  - All reasonable bundle-price ascending auctions will terminate in at most *s*n*max-bid/min-bid-increment* steps.
  - Theorem (Blum, Jackson, Sandholm, Zinkevich): exponentially many demand-oracle queries are needed, when $s=\sqrt{m}$.
- Can item-price auctions be faster than bundle-price auctions?
- Depends what you count as "information" in bundle-price auctions
  - Allow concise representation of bundle prices → at least as powerful as item-price auctions
  - Require to list each non-0 bundle price → yes.

Finding a "hidden" subset

- Consider the following case:
  - $v_1(S) = 2|S|$,
  - Except for a single "hidden" set H with $v_1(H) = 2|H| + 1$.
  - $v_2(S) = (1+\varepsilon)2|S|$, for all S.
- Efficient allocation requires finding H and giving it to 1.
- A Walrasian equilibrium with a price of $(2+\varepsilon)$ per item exists, and can be found quickly by item-price auctions.
- Assume even $|H|=m/2$ is known by a bundle price auction.
- No information about the identity of H can be obtained unless $p_1(S) \geq |S|$ for all sets S of size $m/2+1$.
- But this requires exponentially many bundle prices.

Suggestions so far

- A hybrid approach may be better than either item-price auctions or pure bundle-price auctions.
- Allowing bundle-price auctions to represent bundle prices succinctly suffices.
- How succinctly?
  - Either item prices or bundle prices
  - Arbitrary ORs of sub-bundle prices
  - General formula in some bidding language
  - Most extreme case: the aggregated bid of all others so far
- Try to evaluate auction mechanisms according to what types of valuations they can handle with reasonable information transfer.
- Challenge: a mechanism that can handle any XOR/OR-formula bids (in time polynomial in the bid size).

Approximation Algorithm

For $c = \varepsilon, 2\varepsilon, \ldots, max\text{-}bid$

For each bidder i

Let S_i be the demand (*if not* ϕ) where all item prices are c

If demand became ϕ for the first time allocate S_i to i (taking away any items that were previously allocated)

If (total value of solution found) < $v_i(\{1 \ldots m\})$ for some i then

Ignore current solution and allocate everything to i

Theorem: This gives a min(n,O(√m)) approximation

Theorem: Any auction that gives a better approximation requires an exponential number of queries.

Nisan&Segal

Sub-modular Valuations

<u>Algorithm</u>:

Initialize, $p_{i,j} = \infty$ for all bidders i and items j

For every item j do

Perform Dutch auction on item j; let i be the winner

$S_i \leftarrow S_i \cup j; \quad p_{i,j} = 0$

<u>Theorem:</u> If all valuations are sub-modular then this is a 2-approximation. Lehman&Lehman&Nisan

<u>Theorem:</u> Any auction requires an exponential amount of communication to find an exact solution even for sub-modular valuations. Nisan&Segal

Auctions with Duplicate Items

- Assume that there are *k* units of each good.
- Each bidder wants at most one from each good type – i.e. $v_i()$ is still a functions of sets (rather than multi-sets).

Online Algorithm:

Initialize item prices: $p_1 = \ldots = p_m = v_{min} / (2km)$

For all bidders *i* in order of arrival:

$S_i \leftarrow D_i(p_1 \ldots p_m)$

For all items $j \in S_i$ do

$p_j = \rho \, p_j$ (for some well chosen ρ)

Theorem: This auction is valid, incentive compatible, and gives as good approximation as computationally possible.