Combinatorial Interlicense Competition: SPECTRUM DEREGULATION WITHOUT CONFISCATION OR GIVEAWAYS

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Abstract

The right to use US radio frequency spectrum is extremely valuable. Unlike most property rights, licenses to use radio spectrum are granted for limited terms and carry significant restrictions on how the radio spectrum is used. Economic efficiency suggests that existing license rights should be expanded to give users the flexibility to redeploy spectrum to its most valuable use and to trade licenses or unused capacity on secondary markets. Distributing these expanded rights to use radio spectrum raises questions of both efficiency and equity. This paper proposes an auction mechanism for distributing additional usage rights: combinatorial interlicense competition. Derived from an auction procedure used by the Interior Department for the auction of mining leases, this mechanism grants license relaxation rights using competition, while ensuring that the government still obtains the fair value of the licenses it is granting. Interlicense competition overcomes the fact that the holders of existing usage rights have a strong competitive advantage over any challengers. The combinatorial nature of the auction deals with the fact that combinations of licenses may be more valuable than the licenses are separately. This auction mechanism could be a useful addition to the arsenal of tools available to the FCC. Its availability means that it is not necessary to give away spectrum to incumbents to gain the advantages of fully flexible license rights.

1. Introduction

In the United States, the radio spectrum is legally the property of the public, but with varying degrees of private use rights. The radio spectrum is extremely valuable—many times more valuable than all of the gold in Fort Knox.[1] For over three quarters of a century, the government has been making policy with the aim of having this valuable asset used in the public interest—a nebulous standard that has been subject to many different interpretations.[2] At present, some frequencies are reserved for government uses—defense, air traffic control, public safety, etc.—and some are licensed to companies for a variety of particular uses such as broadcasting and fixed and mobile communications. Almost all the valuable bands of spectrum—those that propagate well through walls, trees, and weather—have already been assigned for some use.[3]

The current system of spectrum management is based on a command-and-control framework. The Federal Communications Commission (FCC) manages the allocation of private and state and local government uses of spectrum, while the National Telecommunications and Information Administration (NTIA) coordinates the federal uses of spectrum. For non-federal uses, traditionally the FCC allocates blocks of spectrum to types of uses, such as broadcasting or satellite, creates channel assignments and then assigns license rights to users. Licenses often specify where, when and how the licensee may use the radio spectrum. For instance, a typical television license will specify a transmitter tower location, height, power levels, channel assignment and broadcast technology.

Licenses traditionally were distributed on a first come basis. When more than one applicant wanted a particular license, the FCC was forced to choose among competing applicants. For most of its history it used comparative hearings, commonly referred to as beauty contests. This became an expensive and inefficient procedure and was replaced with lotteries in the 1980s. In 1994, the FCC began conducting auctions to assign licenses that had mutually exclusive applications. The FCC has pioneered innovative auction formats to assign rights to use radio spectrum. The assignments to date generally have been for bands of spectrum where either there were no significant incumbent licenses or there were clear rules for removing the incumbents.

2. Distributing Expanded License Rights

Currently, the FCC allocates spectrum on a licensed or unlicensed basis. Examples of licensed services are mobile telephone, broadcasting, and direct broadcast satellite. The licensee pays the government or promises to serve the public interest in return for use of the public airwaves. Examples of license-exempt services are cordless phones, garage door openers, Wi-Fi devices, and other consumer devices. On license-exempt bands, consumers share the spectrum without paying a fee to either the government or a licensee. This paper will not address when access to the spectrum should be under a licensed or unlicensed regime. Instead, we take the decision to expand the user rights in a currently licensed band of spectrum as given and look to how those expanded, and hence more valuable, rights are distributed to private entities. There is a general consensus at the FCC and among policy experts that the commercial use of spectrum should be largely deregulated, giving users far greater flexibility to determine the service provided on a band, or even to sell or sublease access to other firms through secondary market transactions.

Many interesting questions are raised in trying to define the scope and nature of the rights that should be attached to licensed radio spectrum. At one extreme are fee simple property rights and at the other extreme are time-limited, royalty-based rights leases. These are important questions, but this paper is agnostic with respect to them. It is concerned with the method of distributing expanded rights, however they are defined.

There are at least two problems inherent in distributing expanded license rights in spectrum. First, there is a desire (or, at least, a political imperative) to respect the current use rights granted to current licensees, including the presumption a license will be renewed, even when those licensees received their licenses free. Indeed, the Communications Act stipulates that licenses are temporary and confer no residual ownership interests. Second, both fairness and efficiency require that the government receive most of the value of the liberalization of the licenses. Since the right to use the spectrum for commercial purposes is worth hundreds of billions of dollars, the fairness aspect of a spectrum giveaway probably requires little comment beyond Senator McCain's observation that "They used to rob trains in the Old West. Now we rob spectrum." However, the efficiency argument is more subtle, and it is critical since the case for "privatizing" the spectrum is based upon efficiency.

The essence of the efficiency argument against a giveaway is that if the government fails to get full value for assets it gives away, the money it does not receive must be raised with taxes.[4] There is a substantial economic literature documenting the marginal inefficiencies associated with raising money from income taxes.[5] A conservative estimate is that for every three dollars in federal revenue forgone (requiring, therefore, additional taxes to be raised) there is an additional dollar of lost productivity. **Consequently, the added cost of the deadweight loss of raising government revenues—or worse, increasing the federal deficit—to compensate for lost spectrum revenue should be recognized as part of the price paid by the public when spectrum rights are given away.**

Proposals exist to distribute spectrum relaxation rights. In the summer of 2002, the FCC established a Spectrum Policy Task Force (SPTF) with the mission to "provide specific recommendations to the Commission for ways in which to evolve the current 'command and control' approach to spectrum policy…."[6] In the end, the SPTF recommend that the Commission find a modest 100 MHz of spectrum below 5 GHz to transition from the current command and control regime to a market-managed regime based on flexible spectrum rights.[7]

The SPTF does not recommend a specific process for distributing the expanded spectrum use rights, but two of the Task Force's members do. FCC senior economist Evan Kwerel and recently retired FCC senior engineer John Williams propose an auction to distribute rapidly significant amounts of spectrum relaxation rights, commonly referred to as the 'Big Bang' auction proposal. Their proposal entices incumbents to put their existing spectrum license rights into the auction so that bidders will be able to bid on the full set of rights for a specific band of spectrum. Incumbent license holders are given three incentives to participate: first, they receive 100% of the auction receipts if their band is sold (or a prorated portion if the band is shared or combined with FCC reserve spectrum); second, if the band goes unsold, the licensee gets the expanded rights for free; and third, the licensees get the right to match any competitive bid and thereby "buy-back," at zero additional cost, the expanded rights (thus discouraging others from competing for the rights). They propose auctioning bands totaling 438 MHz of spectrum under 3GHz.[8] This ambitious auction proposal would likely distribute expanded use rights to incumbents for free or at far below their value. This is consistent with Kwerel and Williams' approach to spectrum management that focuses solely on the efficiency gains associated with distributing the expanded and valuable license rights to the largest amount of spectrum possible as soon as possible.

The likely low revenue outcome of the Big Bang proposal is driven by the presumed ability of incumbents to hold up the use of spectrum by new users. Hold up occurs when the incumbent can demand a disproportionate share of the benefits from the new, higher valued uses of a band of spectrum. By scaring away other bidders, the incumbent becomes the likely only bidder in many bands. It is a bit like trying to sell a valuable block of downtown real estate when someone has the right to have a lemonade stand on it. Who will offer to pay anything near its real value when the owner of the rights to the lemonade stand can block any potential use of the property? (This example is not far fetched. The right to broadcast television on a UHF station in a major city where almost everyone who watches the station gets their signal over cable is probably worth a few percent of what the spectrum would be worth for mobile communications.[9]) Normally, if such downtown real estate were put up for competitive sale, the owner of the lemonade stand rights or someone in partnership with him would be the only serious bidder. With only one bidder, market forces could not be relied upon to set a price that comes anywhere close to the value of what is being sold. The purpose of this paper is to propose a way to overcome this difficulty.

3. Current Examples

To fill out the types of expansion rights we are proposing to be distributed, a few examples may be useful. The rights to be distributed by the auction fall into two broad categories, both of which incumbent licensees can likely effectively block new licensees from using for new higher valued uses. The first is a filling out of the rights in currently licensed portions of spectrum. Examples of this type of expansion right would include:

- <u>Expanded rights for television licensees.</u> These expanded rights would allow television broadcasters to cease television broadcasts and use their licensed spectrum for other uses.
- <u>Multichannel Multipoint Distribution Service (MMDS).</u> MMDS licenses are for fixed wireless uses. Expansion rights would include allowing mobile uses in the band.

A second type of expansion right fills out licensing of currently unlicensed portions of bands allocated to private, licensed uses. Examples include:

- <u>Unused spectrum in the television bands.</u> Broadcast television stations require that spectrum adjacent (both in spectrum and geography) to the licensee not be Therefore, channel 7 in Washington, DC prevents the use for broadcasting of channels 6 and 8 in DC and channel 7 in surrounding areas. The unlicensed portions of the television band could be licensed.
- Unused spectrum in the fixed point-to-point microwave bands. Point-to-point microwave communications leaves much of the spectrum surrounding the links unused. This spectrum could be licensed. (Note that the PCS auctions were for the unused spectrum around existing links and the right to move the existing links to another band—containing features of both types of expansion rights.)

4. An Alternative: Interlicense Competition

We describe an auction procedure that can be used to sell relaxation rights that liberalize the use of spectrum while obtaining for the government the fair value of the licenses it is granting. The heart of the proposal is an adaptation of a procedure suggested by C. Bart McGuire and used in the early 1980s by the U.S Department of the Interior to auction coal rights to Federal coal tracts where the owners of adjacent coal deposits were the only logical bidders.[10] In the context of coal, the approach was called "intertract competition." It made the bidders for different coal tracts compete with each other. This approach was authorized by Congress and evaluated favorably by the "Linowes Commission,"[11] established by Congress to investigate a scandal that shut down Interior Department coal leasing in the early 1980's.

The proposal also draws on other ideas from the auction literature. One is to treat a constraint on the total amount to be sold as "soft." This idea dates back to discussions of "PURPA auctions" for electricity supply contracts.[12] Since to be economic, new services may require combinations of current licenses, the proposal allows bids on combinations of licenses. (However, for ease of exposition, we explain first a noncombinatorial version.)

Under this proposal, no licensee's rights will be damaged or limited in any way. However, under this proposal, no licensee or other party will get spectrum relaxation rights without competition. In particular, current licensees for a service that greatly under utilizes spectrum will have to compete with others to get their license restrictions eased even though they may be the only bidder for the particular rights that complement theirs. **It is not necessary to give away spectrum rights in order to have the advantages of private ownership incentives**.[13]

The next section of this paper presents the interlicense competition proposal, first in simplified form and then in a more complicated form in which bidders can make offers on relaxation rights on combinations of licenses. This is followed by a discussion of the proposal, of implementation issues, and of some specific concerns in spectrum management policy such as public interest obligations.

5. A Simplified Proposal for Interlicense Competition

Here is a simple version of the interlicense competition proposal to expand spectrum license rights without either giving them away for much less than their value or forcing the holders of existing rights to release them (with or without compensation).

Under this simplified proposal, Congress will authorize the FCC to announce an annual or perhaps biannual series of auctions of "overlay" or "relaxation" spectrum rights. Each auction will relax the current regulatory constraints on a given amount of spectrum (measured in units of bandwidth times the population area covered, i.e., in MHz-Pops[14]) for essentially unrestricted use subject to responsibility for noninterference with licenses for other frequencies and other geographic areas as well as any existing license on the spectrum. However, the amount to be sold in a single sale will be a relatively small fraction, perhaps 10% to 20%, of the amount upon which bids will be accepted. While for national security, public safety, or other special purposes some spectrum may be excluded from bidding in these sales, relaxation rights for most privately licensed spectrum will be eligible for sale and sold if the offer for it is high enough. Any currently licensed spectrum offered will be subject to the rights of the current spectrum license holder. Rights to currently unlicensed spectrum will also be included. For example, TV channel 2 in Washington would be included (but subject to the requirement that its use not interfere with channel 2 in Baltimore).

The current license holder may bid to relax the restriction on her license. Others may also bid for these relaxation rights, although other bidders may well be at a disadvantage relative the current rights holder. Similarly, the holder of the rights to TV channel 2 in Baltimore may have an advantage over other bidders for the unlicensed right to TV channel 2 in Washington. The auction will be a sealed-bid, market-clearing-price auction. In this simple version of the auction, there will be no combinatorial bids and spectrum with the highest bids per MHz-Pop will be sold up to the cut off limit on MHz-Pops for the sale. The important consequence of this is that a license holder wishing to relax the constraints on a license will have to compete for the right to do so with holders of other licenses who also wish to relax the constraints on their licenses.

In this simple version of the auction, in order to select the winning bids the FCC will first rank order the bids with respect to the amount offered per MHz-Pop. Starting with the highest ranked bid, the FCC will award eligible bids that do not conflict with previously accepted bids until it reaches a bid that would put the total sold over the limit set in advance of the auction for MHz-Pops. This bid is the marginal bid and will set the price per MHz-Pop for all accepted bids (whether it itself is accepted or not). If accepting the marginal bid would make the total MHz-Pops sold exceed the announced target by less than a pre-announced tolerance percentage, the bid will be accepted. If accepting the bid would result in exceeding this tolerance limit, then the FCC will reject the bid. All bids offering a price per MHz-pop less than the marginal bid will be rejected. If the bid acceptance process ends without reaching the target number of MHz-Pops, then all bids that have not been rejected will be accepted and the price per MHz-Pop will be the minimum allowable bid.

Three easy-to-show theoretical results are worth noting. First, if the tolerance limit exceeds the size in MHz-Pops of the largest license, then the auction will always end with the acceptance of the marginal bid. Second, whether the auction ends with the acceptance of the marginal bid or its rejection, there are no price anomalies; all accepted bids offer higher unit prices than all rejected bids. Finally, whether the auction ends with the acceptance of the marginal bid or its rejection, the total value expressed in the accepted bids is the maximum possible for the number of MHz-Pops sold.

Interlicense Competition: More Design Details

In auction design, the devil is in the details. It is vital that a number of procedural details be set up correctly. Substantial deposits should be required of bidders, and there should be prompt payment by winners and prompt awards to them upon completion of the auction. If citizenship or other qualifications are required, bidders should be required to assert under oath at the time the deposit is made that they meet them. All eligibility challenges except ones connected with criminal prosecutions for perjury should be limited to the period before the auction.[15]

Immediately after the auction, the FCC should return deposits on unsuccessful bids. Successful bidders will pay the remainder of the price of what they have won, and licenses will be awarded to them. If they fail to pay, they will be in default, should lose their deposits and get no rights, bankruptcy laws notwithstanding.

Before each periodic auction, the FCC will announce to potential bidders the geographic units (and their populations) that will be used and any frequencies that are not available. If some frequencies are available in some geographic regions but not others, this too will be announced. For simplicity, we will call the units the FCC announces "licenses." All frequencies not explicitly excluded will be available subject to the specific rights of existing licensees. The FCC will also announce the tentative target total number of MHz-Pops to be sold. Bidders should not be surprised by the announcement since a long-term plan for making frequencies available will have been adopted.

The FCC will also announce the deposit required from bidders per MHz-Pop of bidding eligibility. The deposit will be a substantial fraction of the anticipated price per MHz-Pop in the sale. It may also serve as the minimum bid per MHz-Pop, which should also be a substantial fraction of the anticipated price. In order to avoid a noncompetitive auction, after the deposits are received the FCC will, if necessary, announce a reduced total of MHz-Pops to be sold so that the amount to be sold is no more than some pre-announced fraction, say one-fourth, of the total eligibility.

Lower band and upper band relaxation rights should be sold in separate auctions, because not all MHz-Pops are the same. For example, lower frequencies that are suitable for mobile communications are more valuable than the upper frequencies (above 3 GHz) that do not readily propagate through walls, foliage and precipitation. It is important for the auction that bids be on the same basis so that they can be meaningfully compared. In addition, some further refinements in the $/MHz-Pops based on the frequency of the band in the bid may be considered useful.

Note that each of the periodic auctions can be treated as a one-time, sealed-bid auction. Hence, there is no need to restrict the bids to round numbers to prevent signaling. Since the bidders have the possibility and incentive to use lots of significant digits in their bids, ties should be exceedingly rare. If ties become common, collusion should be suspected. To discourage tacit collusion, bids at the exact same price should be treated as a single bid. If accepting this "bid" would result in too much spectrum being sold, all of "it" should be rejected. This also means that no rule is needed for resolving tie bids.

An Example

Before going further, it may be useful to work through a small example. Table 1 gives the highest nine of a large set of bids. For convenience, the bids have been numbered in decreasing order of bid amount

Table 1: A Set of Bids

Bid #	$/MHz-Pop	License #	MHz-Pops (x10^6)
1	6.0121	4321	60.2
2	5.8327	5432	43.5
3	5.7511	4321	60.2
4	5.6330	6543	12.7
5	5.5112	7654	44.0
6	5.5081	8765	32.6
7	5.0423	9876	25.8
8	4.8899	1234	10.4
9	4.8001	2345	10.9
etc.			

Suppose that the government has announced that it will sell relaxation rights for 200 (x10^6) MHz-Pops with a tolerance of 5%. In this case, it will accept bid 1 and bid 2. It will reject bid 3 because it has already sold the relaxation rights to license # 4321 to bid 1. It will then accept bids 4, 5, and 6. This brings the total MHz-Pops of accepted bids to 183.0 (x10^6). Bid 7, if accepted, would bring the cumulative number of MHz-Pops of accepted bids to 208.8 (x10^6). Since this is within 5% of the target of 200 (x10^6), the bid will be accepted and its price will set the price of all accepted bids at $5.0423 per MHz-Pop. If the tolerance were only 2.5%, bid 7 would cover too many MHz-Pops to accept. It would be rejected, but it would still set the price for all accepted bids. Bids 8 and 9 would not be accepted even though accepting bid 8 would leave the total MHz-Pops sold below 200 (x10^6) MHz-Pops and accepting bids 8 and 9 would leave the total at 203.3 (x10^6) MHz-Pops, below 102.5% of that amount.

In this example, accepting bid 8 or bids 8 and 9 after rejecting bid 7 would create two anomalies. First, a bid has been rejected that would have offered a higher unit price than an accepted bid. Second, a bid below the demand curve would have been allowed to set the price. This second anomaly could be quite large, if for example, there was a very low bid for relaxation rights on a license with just 1 (x10^6) MHz-Pops. It could be accepted in addition to bids 8 and 9 and still leave the total of accepted bids below 205 (x10^6) MHz-Pops.

6. Interlicense Competition with Combinatorial Bids

It is quite possible that the relaxation rights on an FCC license are worth more if the relaxation rights on other licenses are also obtained. This effect could be mild or it could be critical as when a proposed communication service would absolutely require the relaxation rights of many existing licenses. In addition, it is possible that relaxation rights on alternative licenses would allow a proposed service so that a bidder would like to offer bids in the alternative. Finally, it is possible that bidders are capital limited and would like to limit their total expenditures in an auction. Thus, it is potentially quite useful to allow bidders to bid for combinations of relaxation rights rather than just for individual rights and to place constraints on their bids. However, allowing bids on combinations and such constraints makes selecting the winning bids more difficult, something we must deal with.

We do not know how to deal "optimally" with these trade offs, but we can propose an auction form that deals reasonably with them. In the presence of the synergies that exist, it generally is better than allowing no bids on combinations. The basic idea is simple. Bidders may make combined bids for different licenses. For example, a bid may offer $4.00 per MHz-Pop for license 1 and license 2. Because this is a combinatorial bid, it is not separate offers to buy either license 1 or license 2 at $4.00 per MHz-Pop; it is just an offer for the combination.

The following example may be instructive. Two items of the same size, A and B are for sale. Bidder 1 values A at 3, B at 3, but the combination of A and B at 9 –4.5 per unit for both items. In the absence of combinatorial bidding, her only safe course is to offer 3 for A and 3 for B. If she offers more, she risks winning one and not the other and suffering a loss. In the proposed auction, she can offer a bid of 3 for A, a bid of 3 for B and a bid of 9 for the combination of A and B. If the best competitive bids are 4 for A and 4 for B, her bid of 9 will win both. However, if the best competitive bids are 1 for A and 8 for B, only her bid of 3 for A will win. Note that if she had not bid 3 for A, her bid of 9 for A and B would have won. Thus, to some extent, she is bidding against herself and will have incentives to act strategically.

Here is the specific auction we propose. It is a one-shot, sealed bid auction. In it, each bid is a price per MHz-Pop for the relaxation rights to a particular set of FCC licenses covering given geographical areas and ranges of frequencies. Each bid is made subject to several possible constraints by the bidder. The first such constraint is the eligibility constraint. Based upon her deposit, each bidder is limited to a given number of MHz-Pops. This does not constrain the number of MHz-Pops that she can bid upon, but it does constrain the number she can win. If her deposit is large enough, this constraint will not be binding.

The second kind of constraint is a budget-like constraint specified by the bidder to apply to the sum of her successful bids (not, however, the market-clearing price that the bidder would have to pay). This constraint prevents her from winning relaxation rights that will, in total, cost more than she is willing to spend. Its use by a bidder is voluntary. Bidders would probably prefer to have a budget constraint on actual expenditures, but this might cause computational difficulties. Since budget constraints are often "soft" constraints, this kind of budget constraint may prove useful. It would allow bidders freedom to bid on many different alternatives. In its absence, bidders might find their managements imposing more drastic "exposure" constraints on their total bids as is common in simultaneous sealed-bid auctions for offshore oil rights. (See Rothkopf 1977.)

The third kind of constraint is an "exclusive or" or alternative constraint. A bidder can always make two bids mutually exclusive by including the same license in both. Thus, if a bidder has a bid for relaxation rights for licenses A and B, and another bid for relaxation rights for licenses B and C, both bids cannot win since relaxation rights on license B can only be sold once. Thus, the bids will, in fact, be treated as bids for {A and B} or {B and C}. In addition, the FCC could and should allow each bidder a limited number of "pseudo-items" it can bid on.[16] By including such a pseudo–item in two different bids, a bidder can make the two bids mutually exclusive even though they don't actually overlap. Thus, for example, a bid for A, B and pseudo-item 1 is in conflict with a bid for C, D and pseudo-item 1. The use of such alternative constraints will allow bidders to attempt two or more independent ways to reach a given goal.

In addition to these bidder-specific constraints, the selection of winning bids is constrained by the (soft) limit on the number of MHz-Pops to be sold and by the requirement that the relaxation rights on each license either be sold once or not sold at all. We propose that subject to all of these constraints, the FCC select the bids that maximize the value expressed in the bids it accepts.[17] This mathematical problem is stated formally in the Appendix. It is an integer programming problem, which implies that it is in a class of problems that are potentially computationally difficult.[18] However, just as in the simplified problem discussed above, we avoided exact solution of the problem initially faced, we plan to avoid this problem here. As discussed below, we plan, instead, to solve a series of linear programming problems. Unlike integer programming, linear programming problems are not in the potentially computationally difficult class of problems.

One computational concern deserves special mention. With this auction form, bidders may have little to lose by submitting many slightly different conflicting bids. If the FCC anticipates the total number of bids posing a computational problem, it can require a nominal processing charge with each bid. This will not inhibit any serious bids, but could head off computational problems. In addition or instead, the FCC could impose a generous limit on the total number bids a bidder could submit. This would let a bidder express all of her important values.[19]

Selecting the Winning Bid

We will now describe the general computational procedure for selecting the winning bids and setting the market-clearing price for a given set of bids. After we have done this, we will give a simple example.

The solution procedure begins by solving the integer programming problem given in the Appendix as a linear program. That is, the problem of maximizing value reflected in accepted bids is solved ignoring the constraints that force variables to take on integer values. We will call this Step 1. If this calculation happens to find a solution in which all of the variables are, in fact, integers, the solution also solves the integer programming problem and is accepted as the solution to the bid acceptance problem. The lowest unit price (in $ per MHz-Pop) of any accepted bid is used to set the unit price of the relaxation rights.

If, as is quite likely at first, some integer constraint is violated, the procedure then goes on to modify the problem. This is Step 2. If the budget constraint of a bidder is binding and this results in the proposed sale to that bidder of a fraction of the relaxation right on a license, that budget constraint is tightened to eliminate the fractional purchases. If more than one bidder is in this situation, all of their budget constraints are tightened so that none of them are buying a fractional part of a relaxation right. Similarly, if the eligibility constraint of one or more bidders is binding and this results in the proposed sale to that bidder of a fraction of a relaxation right on a license, the eligibility constraints of those bidders are tightened to eliminate the fractional purchases. It is appropriate to make all of these changes simultaneously since reducing the fractional purchase of one bidder with a binding budget or eligibility constraint, will not eliminate a fractional purchase by any of the others. The calculation then returns to Step 1.

If no budget or eligibility constraint results in the purchase of a fractional relaxation right, then in Step 3 the calculation checks to see if relaxing the constraint on the total MHz-Pops to be sold, but not beyond the pre-announced tolerance limit, will result in the sale of all of those rights and do so without violating a budget-like constraint or an eligibility constraint. If so, the constraint on the total MHz-Pops to sell is relaxed in order to make that sale. The calculation is then complete, all of the bids in the optimal solution are accepted, and the price per MHz-Pop for all sold rights is set by the price of this marginal bid. If relaxing the constraint on the total number of MHz-Pops to the tolerance limit results in the violation of the budget-like or eligibility constraint of the bidder who made the offer on the marginal license(s), that offer is eliminated and we return to Step 1. If no budget or eligibility constraint is violated but the maximum relaxation still leaves the marginal offer or offers only partially filled, then all of the marginal offers are rejected, but their unit price is used to set the price for all accepted bids.[20] All of the other bids in the optimal solution without the relaxation are accepted.

Figure 1 illustrates this process.

Computer scientists may wish to note that since each time the procedure returns to Step 1 at least one bid is permanently discarded, the number of linear programming problems that must be solved is bounded above by the number of bids. Since the worst case bound for computational effort for solving linear programming problems is polynomial, so is the worst case bound on the total amount of computations involved here if the number of bids is limited. Worst-case bounds are usually conservative. In this case, they are likely to be extremely conservative. The reason is that what is being sold, in most cases, is relaxation rights on existing licenses. The value of these rights should be higher to the holder of the existing license or to someone with whom she strikes an exclusive deal. Hence, competitive conflicting combinations from *different* bidders should be rare.

An Example

We now present a highly simplified illustrative example. It involves relaxation rights on the 18 licenses shown in Table 2.

Table 2: Licenses for the Example

License #	MHx-Pops ($x10^6$)
1	60.2
2	43.5
3	60.2
4	43.5
5	44.0
6	32.6
7	25.8
8	37.4
9	10.9
10	48.7
11	30.8
12	51.9
13	10.2
14	8.7
15	18.3
16	43.8
17	82.0
18	64.2

These licenses have a total of 715.7 million MHz-Pops. We will assume that the sale will try to sell 170 million, approximately one quarter of them, and have a 10 percent tolerance, thus allowing sale of up to 187 million MHz-pops. We assume that there is a minimum bid of $.01 per MHz-Pop, and we further assume that there are 12 potential bidders.

Bidder 1 controls licenses 3 and 4 and would like to change the service offered on them. Doing so on 3 alone is feasible, but doing so on 4 alone is not. His values for relaxation rights are $100 million for 3 and $125 million for licenses 3 and 4. He will make two bids: $100 million for 3, and $125 million for 3 and 4.

Bidder 2 has conditional deals with the holders of licenses 6, 7, and 8. She needs to get rights to just one of these to provide a new service. Her value is $25 million for 6, $20 million for 7, and $30 million for 8. She will make bids in these amounts for the licenses and a "pseudo-item," P2, so as to be sure not to win more than one.

Bidder 3 controls licenses 1 and 2. She currently has no plans to change the service she is offering on them, but would like to lock in future flexibility. She is willing, independently, to pay $10 million for relaxation rights on 1 and $8 million for relaxation rights on 2 and will make separate bids of these amounts.

Bidder 4 is a "bottom-fishing" speculator. She controls no licenses. She decides to bid \$1 million each on licenses 1 through 3, 5 through 8, and 10 through 12, but she wants to be sure not to spend more than \$3 million dollars. Therefore, she will link her ten bids by a budget-like constraint of \$3 million. To avoid having her bids for licenses 1 and 3 (which happen to cover the same number of MHz-Pops) tied, and thus treated as linked, she will add \$0.01 to her bid for license 1.[21] In addition, she can only raise enough up-front money to cover deposits for 170 MHz-pops. Hence, she realizes that if she were high bidder on licenses 1, 3 and 12, her bid on license 3, the one with the lowest bid per MHz-pop, would be rejected. She also notes that her bids, while low, meet the minimum bid requirement.

Bidder 5 controls license 5, is willing to pay \$8 million dollars for relaxation rights, and will bid this amount.

Bidder 6 controls licenses 10, 11, and 12. He wants to offer a service that will require two of the three licenses and would benefit from the third. His values are \$100 million for 10 and 11, \$130 million for 10 and 12, \$110 million for 11 and 12, and \$150 million for all three. He submits four conflicting bids. The three bids for pairs of licenses reflect his values. However, because the bid for all three licenses covers 131.4 MHz-pops, he thinks it has a significant chance of being the marginal bid. Hence, he shades it and offers only \$140 million for it.

Bidder 7 want licenses 13, 14, and 15. She can pay \$1.02 per MHz-pop for each of them and can pay a slight premium, \$1.04 per MHz-pop if she gets all three. She bids this unit price plus differing small amounts on the three licenses and a unit price of \$1.04 on the three of them.

Bidder 8 and ***bidder 9*** both have licensed spectrum that abuts currently unlicensed license 16 and both want license 16. Bidder 8 offers \$70.3 million for it, and bidder 9 offers \$81.2 million for it.

Bidder 10 wants to keep his options open on his operation on the compliment of license 17. He doesn't particularly want to buy it, but it will bid \$20 million to make sure that someone doesn't win it (cheaply) and then be able to block his future plans.

No one bids on licenses 9 and 18.

Table 3 shows the bids.

Table 3: Bids on Licenses in Table 2

Bid #	Bidder	Licenses	Amount (10^6\$)	\$/MHz-Pop	Budget Constraint
1	1	3	100	1.661	-
2	1	3,4	100	1.205	-
3	2	6,P2	25	0.767	-

4	2	7,P2	20	0.775	-
5	2	8,P2	30	0.802	-
6	3	1	10	0.166	-
7	3	2	8	0.184	-
8	4	1	1+	0.017	B4
9	4	2	1	0.023	B4
10	4	3	1	0.017	B4
11	4	5	1	0.023	B4
12	4	6	1	0.031	B4
13	4	7	1	0.039	B4
14	4	8	1	0.027	B4
15	4	10	1	0.021	B4
16	4	11	1	0.032	B4
17	4	12	1	0.019	B4
18	5	5	8	0.182	-
19	6	10,11	61.3	1.258	-
20	6	10,12	62.9	1.292	-
21	6	11,12	41	1.330	-
22	6	10,11,12	55.6	1.065	-
23	7	13	10.4	1.020	-
24	7	14	8.9	1.020+	-
25	7	15	18.7	1.020++	-
26	7	13,14,15	39.1	1.050	-
27	8	16	70.3	1.605	-
28	9	16	81.2	1.854	-
29	10	17	40	0.488	-

The winner determination calculation proceeds directly to Step 3 since no eligibility or budget constraints are binding. In it, bid 28 for license 16 by bidder 9 is honored as is bid 1 for license 3 by bidder 1. Bid 21 by bidder 6 is the marginal bid, and it fits (barely) within the tolerance so it is honored. It sets the price at $1.33/MHz-Pop. Thus, bidder 9 pays $58.254 million for the 43.8 million MHz-Pops of license 16, and bidder 1 pays $80.066 million for the 60.2 million MHz-Pops of relaxation rights on license 1. Bidder 6 pays his bid of $100 million for licenses 11 and 12 and their 82.7 million MHz-Pops. Thus, the sale takes in $238.32 million for the 186.7 million MHz-Pops of licenses 3, 11, 12 and 16. All unsold relaxation rights will be offered again in next year's auction.

Note that because this is a simplified example involving only 18 licenses and 29 bids, some bidders had reason to think that their bids had a serious chance of being the marginal bid. However, in a large auction involving hundreds or thousands of licenses, shading a bid significantly from value in order to have a positive gain if it is the marginal bid will be an unattractive option for a bidder. The chance that a significant shading will add to profit will be dwarfed by the chance that it will lead to a profitable bid being rejected. Thus, in such large auctions, shading should be minimal. As noted above, a more significant incentive issue may involve bidding on combinations and on parts of the combination. A bid on a part of a combination might combine with a competitor's bid for the other part to best a bidders bid on the entire combination. This could involve some significant strategizing by bidders, but it is not clear that any workable proposal could avoid this.[22]

7. Policy Discussion

Radio spectrum is a highly valuable public asset. There are strong arguments that U.S. spectrum is badly under used and over restricted and that a licensing system based upon expanded and flexible use rights would work better. While there is a legitimate need to protect temporarily non-licensees who have invested in equipment, the overriding picture is one of misallocation and of the use of administrative procedures to block competition. The proposal in this paper would gradually make spectrum available on a property-rights-like basis. We believe its gradual nature is an advantage. It will take a while for capital markets, and physical ones, to adapt, and non-licensee purchasers of equipment will have a chance for their past equipment purchases to be depreciated. **The use of competition rather than an administrative determination to decide which spectrum is freed up first will tend to assure that the spectrum first released from usage restrictions goes to meet the most pressing unmet needs.**

One interesting perspective on spectrum rights comes from natural resource management. There is a long tradition in U.S. natural resource management of preventing speculative holding of publicly owned resources. This is often done through diligence requirements. Of course, one important difference between land or minerals and radio spectrum is that the lost value from unused spectrum is lost forever—it is a nondepletable natural resource. Nevertheless, there is precedent for the government being the custodian of a natural resource and holding on to ownership (in this case, the relaxation rights to spectrum) until the resource can be used productively.

In choosing an auction mechanism, the government faces two competing goals. On the one hand, the sooner a fuller set of spectrum rights are in private hands, the sooner they can be put to use (within the constraints on the ability of that spectrum to be used productively) with the concurrent increase in consumer welfare. On the other hand, the government wants to receive compensation for the public in return for distributing the valuable relaxation rights to the spectrum. Unfortunately, these two goals are somewhat in conflict. That is, increasing the supply of relaxation rights decreases per unit prices the government will receive. Ideally, this trade-off is solved by transferring the relaxation rights to the private sector at a pace that equates the marginal cost to society in lost service from holding back a little more of the relaxation rights with the marginal cost to society of lost government revenues from slightly increasing the pace that the relaxation rights are distributed. That, rather than giving away the rights, is the efficient course.

The above trade-off illuminates an essential difference between the approach taken in this paper and the one proposed by Kwerel and Williams. Their approach does not consider the marginal cost of government revenues. Supporters of their proposal may think this cost is not relevant for the analysis of the efficient use of spectrum license rights, or they may believe that the optimal trade-off between revenues and the speed of distribution of expanded license rights falls heavily on the side of the distribution of those rights. A third possibility is that may apply to some supporters of the big bang approach is that they primarily care about reducing the size and scope of government by stripping it of resources. In any case, we disagree. As noted earlier, the marginal cost of a lost dollar of governmental revenue has been estimated conservatively at $0.33. This implies that the measured inefficiencies in the use of spectrum from slowing the pace of distribution of relaxation rights can get as high at 33 percent before they outweigh the revenue enhancing effects of that slower pace of spectrum rights distribution.

The auction is not "optimal," but it is reasonable, and in this rich context, what is optimal is not known. It should prove to be workable for fairly large auctions. It should allow bidders to represent important synergies. It should give good incentive signals to bidders whenever the chance that a given bid will be the marginal one is small. It should be relatively resistant to collusion. It should work particularly well in a situation in which on each license one party already has the ability to prevent others from productively using the relaxation rights and thus is the only party bidding for those rights.

In general, the process should efficiently pick out to sell first the most valuable rights. No administrative determination will be needed. Nonetheless, critical spectrum that should not be offered can be protected.

The auction allows combinatorial bidding. Nonetheless, computational problems are avoided by placing mild constraints on the bidding.

The proposed process is independent of the application of the funds it generates. If desired, some of the funds can be used to compensate for dropped public interest obligations. Moreover, the interlicense process is also neutral with respect to the duration of rights auctioned. Congress could determine that the auctioned rights are permanent, or could determine that a spectrum user fee (or lease fee) should attach after the initial license term.

In the past, the FCC has received less for some licenses than it might have because independent companies formed coalitions before entering the auction. This happened to an extreme extent in some European spectrum sales. Hence, it is tempting to suggest that legislation enabling the auctions should protect competitiveness by restricting joint bidding, not just by coalitions formed after bid deposits have been made, but also by joint ventures formed after the legislation is introduced. However, some new uses of spectrum may well require rights held by different parties. In such cases, coalition formation is natural and can be helpful. The solution is for the FCC to limit the amount of spectrum to be sold so that there is a high "eligibility ratio,"-- i.e., there are four or more serious bidders for each license that is to be sold. This should ensure that there is serious competition even in the face of coalitions and discourage coalitions that would pay off only by reducing competition in order to lower prices.

We are not political experts, but we are aware that there is political opposition to giveaways and that many years of advocacy for liberalization of spectrum restrictions has had only modest results. Hence, we believe that those favoring such liberalization could gain political traction towards their goal by supporting this proposal and gaining the support of those opposed to giveaways.

In summary, with interlicense competition no licensee's current rights will be damaged or limited in any way, but no licensee or other party will get spectrum rights without serious competition and some payment back to the public. In particular, those with rights for a use that greatly under uses spectrum will have to compete with others to get their license restrictions eased even though they may be the only bidder for the particular rights that compliment theirs. **It is inefficient and unnecessary to give away spectrum rights in order to have the advantages of private ownership incentives completely flexible license rights.**

Appendix

This Appendix present a mathematical formulation of the optimization problem discussed above in Section 6. It assumes that all bids below the minimum allowable bid have already been deleted. It also assumes that only authorized bids on pseudo-items are included and that the size of each pseudo-item is defined as 0.

Let i index licenses, j index bids, k index bidders, and c index combinatorial bids. If bidder k makes bid j on license i, let b_{ijk} be the price per MHz-Pop offered by that bid. Let x_{ij} be 1 if bid j covers license i, and 0 otherwise; and let X_{jk} be 1 if bid j is by bidder k, and 0 otherwise. Let s_i be the size of license i measured in MHz-Pops. Let y_j be the fraction of bid j that wins; these are the decision variables over which we are optimizing. Let S be the number of MHz-Pops that are scheduled to be sold. Let E_k be the eligibility of bidder k. Let B_k be the budget-like limit of bidder k (infinite if bidder k specifies no budget-like limit).

The optimization problem is

$$
\begin{aligned}
\text{Maximize}\quad & Z = \Sigma_{ijk} b_{ijk} s_i x_{ij} y_j \\
\text{Subject to}\quad & \Sigma_{ij} s_i x_{ij} y_j \le S, \\
& \Sigma_j x_{ij} y_j \le 1, \text{ for all } i, \\
& \Sigma_{ij} s_i x_{ij} X_{jk} y_j \le E_k, \text{ for all } k, \\
& \Sigma_{ij} s_i x_{ij} X_{jk} b_{ijk} y_j \le B_k, \text{ for all } k, \\
& 0 \le y_j \le 1, \text{ for all } j, \\
& y_j \text{ integer, for all } j.
\end{aligned}
$$

The first constraint assures that the relaxation rights to no more than the allowed number of MHz-Pops is sold. The second set of constraints assures that the relaxation rights to no license is sold more than once. The third set of constraint assures that no bidder exceeds her eligibility. The fourth set assures that no bidder exceeds her budget. The fifth set of constraints assures that the fraction of each bid that is accepted lies in the range [0,1]. (Because of the second set of constraints, the upper bound in this set of constraints is redundant.) The final constraint changes what would otherwise be a linear programming problem into an integer programming problem by forcing the fraction of each bid accepted to be either 0 or 1.

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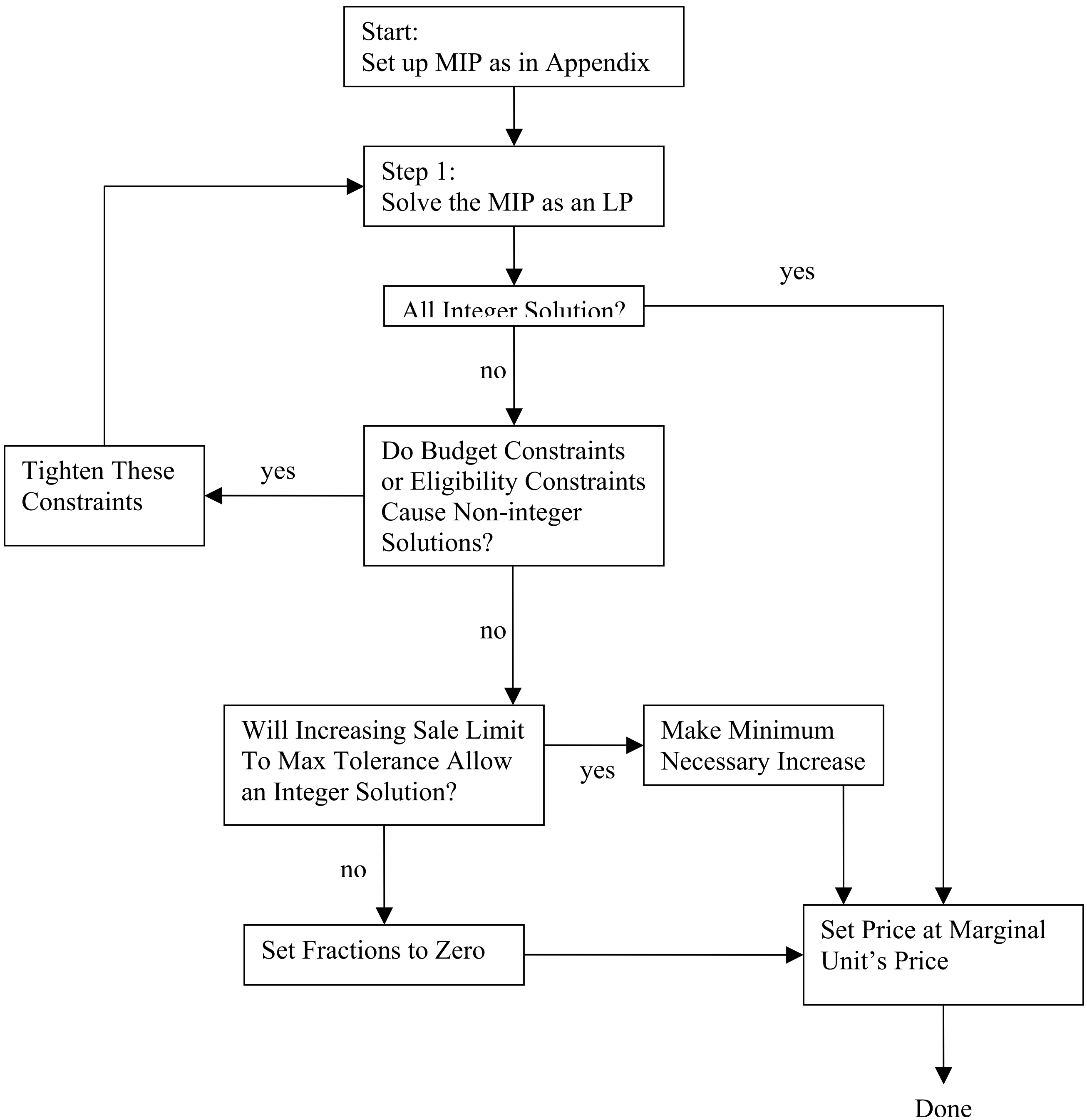
Figure 1

Bid Evaluation Algorithm



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ENDNOTES

[1] *See* J.H. Snider, *et al*., "The Citizen's Guide to the Airwaves," The New America Foundation, Washington, DC, July, 2003.

[2] For a detailed discussion of U.S. spectrum management, see Congressional Budget Office, *Where Do We Go From Here? The FCC Auctions and the Future of Radio Spectrum Management*, Washington, D.C.: U.S. Government Printing Office, April 1997.

[3] For details on the current spectrum allocations, see Bennet Kobb, *Wireless Spectrum Finder: Telecommunication, Government and Scientific Radio Frequency Allocations in the U.S. 30 MHz to 3000 Ghz,* (New York: McGraw-Hill, 2001).

[4] *See* Michael H. Rothkopf and Ronald M. Harstad, "Reconciling Efficiency Arguments in Taxation and Public Sector Resource Leasing," RUTCOR Research Report #66-90 and School of Business Working Paper No. 155, Rutgers University, New Brunswick, NJ. 1990.

[5] While there may be unused opportunities to tax pollution or other externalities, these are likely to be relatively small, and the marginal source of tax revenue is the income tax. For details see Charles Ballard *et al*., "General Equilibrium Computations of the Marginal Welfare Costs of Taxes in the United States," American Economic Review 75, (1985): 128-138. Also see Don Fullerton, "If Labor is Inelastic, Are Taxes Still Distorting?" Working Paper, University of Virginia, 1998. Also see Charles Stuart, "Welfare Costs Per Additional Tax Dollar in the United States," Am. Econ. Rev. 74 (1984): 352-362.

[6] *See* FCC Spectrum Policy Task Force, "SPTF Report," ET Docket No. 02-135, Nov., 2002, p.1.

[7] *See* "SPTF Report," p.51.

[8] *See* Evan R. Kwerel and John R. Williams, "A Proposal for Rapid Transition to Market Allocation of Spectrum," OPP Working Paper No. 38, Federal Communications Commission, Washington, DC, 2002.

[9] *See* Evan R. Kwerel and John R. Williams, "Changing Channels: Voluntary Reallocation of Spectrum," OPP Working Paper No. 27, Federal Communications Commission, Washington, DC, 1992. Also see Snider, *et al*., "The Citizen's Guide to the Airwaves."

[10] *See* U.S. Department of the Interior, Office of Policy Analysis, "Improvements to the Federal Coal Leasing Program Linked to the Use of Intertract Bidding," Report, April 1981.

[11] *See* David F. Linowes, Chairman, *Report to Congress: Commission on Fair Market Value Policy for Coal Leasing*, (1984): 216-222.

[12] *See* Michael H. Rothkopf *et al*., "Designing Purpa Power Purchase Auctions: Theory and Practice," *Competition in Electricity: New Markets and New Structures*, James Plummer and Susan Troppman, Eds., Public Utilities Reports, Inc., Arlington, VA, (1990): 139-194.

[13] "Efforts to extract gains from licensees … should not be permitted unduly to hinder or delay realization of the public benefits from promoting greater competitiveness through spectrum liberalization." Gregory L. Rossten and Thomas W. Hazlett, "Comments of 37 Concerned Economists," Comment on WT Docket No. 00-230, FCC, 2001, p.6.

[14] The units here may be unfamiliar to some. Dollars per MHz per Pop is the same as dollars per MHz-Pops. Both refer to the per capita cost of 1 MHz of spectrum. However, MHz-Pops, which are appropriate here, refer to the amount of bandwidth (MHz) multiplied by the population in the geographic area of the license.

[15] The purpose of this proposed procedure is to prevent competitors of the service to be offered by the new licenses from delaying their competition.

[16] The idea for doing this is due to Fujishima *et al.* 1999.

[17] An alternative, which we do not endorse, would be for the FCC to select the set of bids that would maximize the revenue it receives. Doing so could provide strong incentives for undesirable strategic bidding. In addition, it might lead to the FCC rejecting bids in order to increase the revenue from the sale by preventing the marginal price from falling. We believe that the public will be served best if the FCC makes and sticks to an overall judgment on the best pace at which to release spectrum from regulation taking into account both the efficiency gains from public revenue (which will remove the need for an

equivalent amount of taxation) and the ability of industry to finance and make available services to the public and the public's readiness to make use of these new services.

[18] Some mathematical problems have solution algorithms that even in the worst case grow in length as the size of the problem is increased no more than a given polynomial bound. Such problems are usually considered computationally easy. On the other hand, some problems have no known algorithm that is guaranteed for the worst case to grow no more than a polynomial bound as the size of the problem increases. Large instance of these problems are potentially unsolvable. Integer programming is in the latter class of problems. See Rothkopf, Pekec and Harstad 1998 for a discussion of this in the context of combinatorial auctions.

[19] See Park and Rothkopf 2001 for a discussion of the effect of limiting the number of combinations a bidder in an FCC auction can make and for a report of a related experiment.

[20] In effect, if there are multiple bids at exactly the same unit price on the margin, these bids are treated as a single bid. If this single bid fits within the tolerance limit all of its components are accepted. If not, all are rejected. The reasons for rejecting the marginal bids and not going on to lower bids is the same as the reasons discussed above in the context of the simplified auction. The reason for rejecting all marginal bids if they collectively exceed the tolerance limit is that bidders are free to use many significant digits in their bids. Thus, bids by separate bidders offering the exact same price are suggestive of collusion, and rejecting such bids is a good idea. Bidders can avoid equality in their own bids by adding or subtracting a different tiny amount to each bid, so there should be no problem in rejecting bids from one bidder with the same unit price. This also eliminates any arbitrariness in dealing with ties.

[21] For simplicity of exposition, we are using round numbers for most bids. In practice, bidders would have an incentive to avoid round numbers to avoid unintended ties and to make their bids unpredictable to their competitors.

[22] In particular, Vickrey-Clarke-Groves procedures, which would do this in theory under some circumstances, are not practical. See Hobbs et al. 2000 and Sakurai et al. 1999.