COMBINENET
[DECISION-GUIDANCE SYSTEMS]

Winner Determination in Combinatorial Exchanges

Tuomas Sandholm
Associate Professor
Computer Science Department
Carnegie Mellon University
and
Founder, Chairman, and Chief Technology Officer
CombineNet, Inc.

Outline

- CombineNet company overview
- Performance on real-world combinatorial procurement auctions
- Exchange formulation & problem hardness
- Exchange instance generator
- Experiments with different solution technologies & instance types
- Factors affecting problem difficulty
- Discussion of the expected FCC exchange model

CombineNet, Inc.

- **Leading vendor of markets with expressive competition**
- **Technology development started 1997**
- **Company founded April 2000**
- **55 full-time employees and 9 professors**
  - Tuomas Sandholm, Subhash Suri, Egon Balas, Craig Boutilier, John Coyle, Holger Hoos, George Nemhauser, David Parkes, Rakesh Vohra
- **1 patent issued and 13 pending**
  - Bidding languages
  - Market designs
  - Algorithms
  - Preference elicitation
  - Methods around basic combinatorial bidding that make it practical
  - …
- **Headquartered in Pittsburgh, with offices in London, San Francisco, Atlanta, Brussels**

CombineNet event summary (latest 2 years)

- **~100 combinatorial procurement auctions fielded**
  - Transportation: truckload, less than truckload, ocean freight, air freight
  - Direct sourcing: materials, packaging, production
  - Indirect sourcing: facilities, maintenance and repair operations, utilities
  - Services: temporary labor
  - …
- **Total transaction volume: $6 B**
  - Individual auctions range from $8 M to $730 M
- **Total savings: $1.02 B**

CombineNet applied technologies

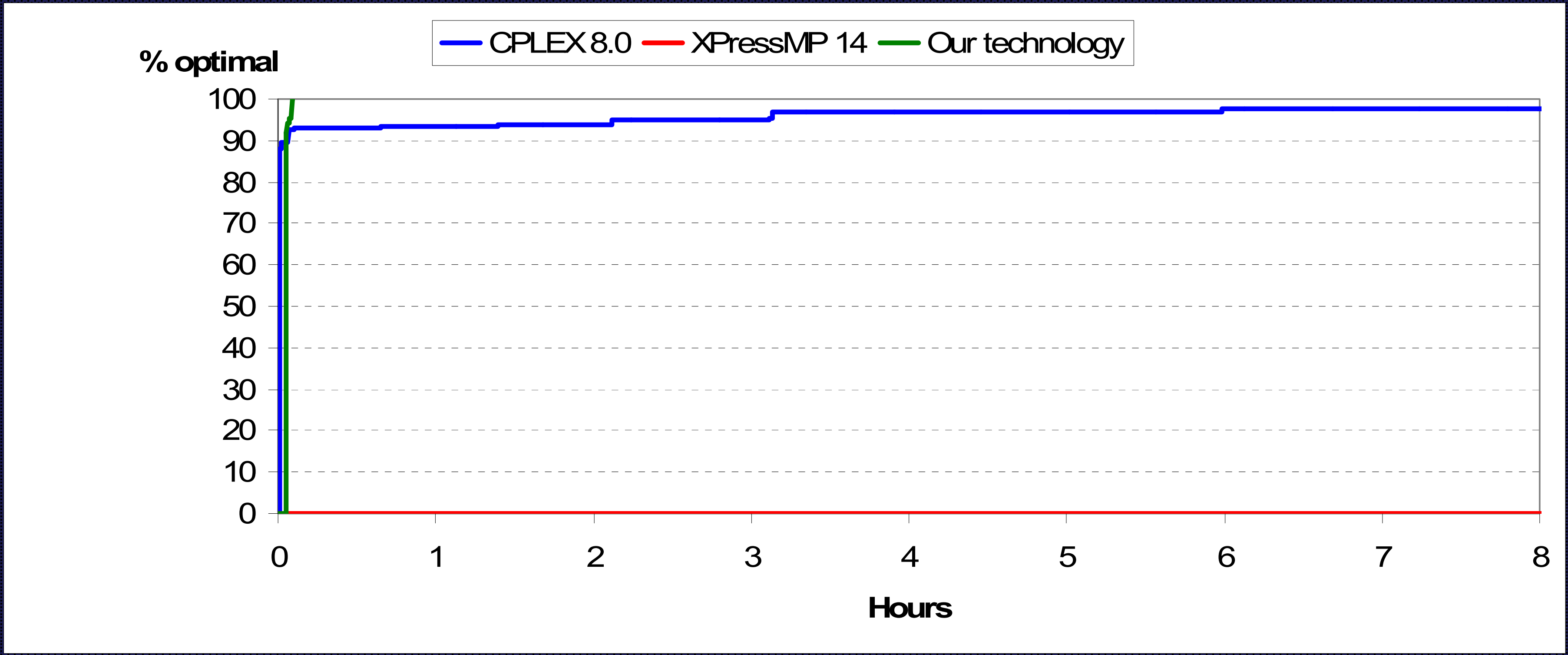
- **Operations research**
  - LP relaxation techniques
  - Branch and bound, Branch and cut
  - Multiple (efficient) formulations
  - …
- **Artificial intelligence**
  - Search techniques
  - Constraint propagation
  - …
- **Software engineering**
  - Modularity supports application of most appropriate solving techniques and refinements, some of which depend on problem instance
  - C++ is effective (fast) implementation language, STL is indispensable
  - XML is effective (extensible) input/output metalanguage
    - Off-the-shelf XML parsers are too slow and heavy for large (100s of MB) inputs, so we built our own
  - …

Largest expressive competition problem we have encountered

- **Transportation services procurement auction**
- **~ 3000 trucking lanes to be bought, multiple units of each**
- **~ 120,000 bids, no package bids**
- **~ 130,000 side constraints**
- **CPLEX did not solve in 48 hours**
- **Our technology clears this optimally & proves optimality in 4½ minutes**
  - Significant algorithm design & software engineering effort 1997-2003

One of the *hardest* expressive competition problems we have encountered

- Transportation services procurement auction
- 22,665 trucking lanes to be bought, multiple units of each
- 323,015 bids, no package bids
- 8 max winners constraints (overall & regional)



Combinatorial *exchanges*

Combinatorial exchanges are a key effort at CombineNet

- **CombineNet has ~40 engineers, almost half of whom work on winner determination technology**
- **The main backend hosted product, ClearBox, does combinatorial auctions, reverse auctions, and exchanges**
  - With hundreds of types of side constraints
  - With multiple attributes and a fully expressive language for taking them into account
- **$1.84 M NIST ATP grant for a 3-year effort for speeding up combinatorial exchanges**
  - One year completed
- **Fastest engine (by 1-2 orders of magnitude) for clearing combinatorial exchanges**

Exchange model formulation

(simple formulation without side constraints shown)

$$\max \sum_{j \in B} p_j x_j$$ = **surplus (alternatively, could maximize liquidity)**

such that

$$\sum_{j \in B} q_{ij} x_j \leq 0 \quad \forall i \in I$$

where

I is the set of items i

B is the set of bids j

x_j is the (binary) decision variable for bid j

p_j is the price of bid j

q_{ij} is the quantity of item i in bid j

quantities are positive for demand, negative for supply

Sandholm ICE-98, AAAI-99 workshop on AI in Ecommerce, AGENTS-00, CI-02

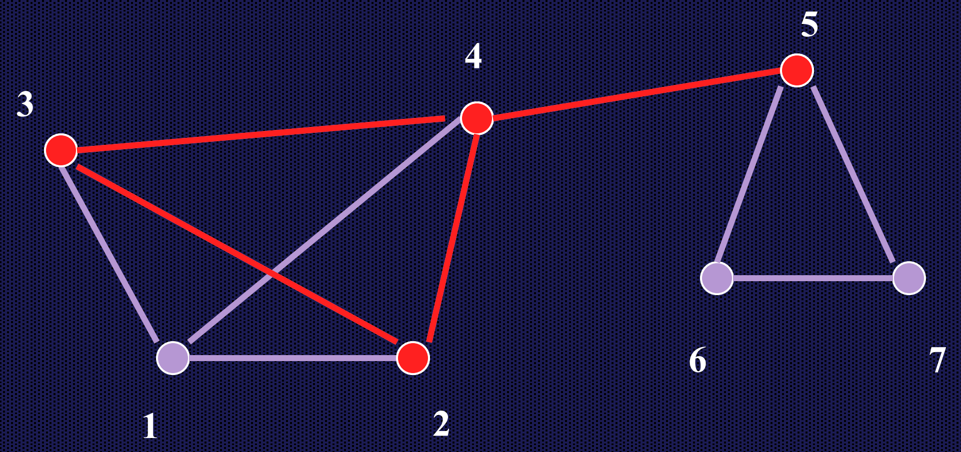
Sandholm & Suri AAAI-00, AIJ-03

Exchange problem hardness
[Sandholm, Suri, Gilpin & Levine AAMAS-02]

- Thrm. NP-complete
- Thrm. Inapproximable to a ratio better than $\#bids^{1-\varepsilon}$
- Thrm. Without free disposal, even finding a feasible (non-zero trade) solution is NP-complete

Exchange instance generator

- **Model of item co-occurrence: building a bundle for a bid**



- **Each bidder has his own subgraph of items**
- **Each item in a bidder's subgraph is only bought *or* sold by that bidder**
- **Complementarity in bids and substitutability in asks determined by edges between items in bundle**
  - Edges assigned weights, sum of weights on a node's edges provides factor used in calculation

Example of pricing bundle bids in the instance generator

- **Items in the bundle**

	2,	3,	4, and	5
– Bidder action	Buy	Buy	Sell	Sell
– Item quantity (α = 0.6)	3	1	4	1
– Market Price	2.34	9.01	6.53	0.14
– Bidder's Price (+/- 25%)	- 5%	+ 7%	+21%	-16%
	2.23	9.64	7.90	0.12
– Bid Price (+/- 3%)	- 1%	+ 1.5%	+ 2.5%	- 1.5%
	2.21	9.78	8.03	0.12
– Graph factor	+ 2%	+ 2%	- 3%	- 1%
	2.25	9.98	7.79	0.12

– Final Price = -14.55 = 3 * 2.25 + 1 * 9.98 - 4 * 7.79 - 1 * 0.12

- **Ask bid at $14.55**

Exchange experiment setup

- **Basics about instances**
  - 50 items, 10 bidders, 50 bids per bidder (= 500 bids)
  - Each bid must be accepted all or nothing
  - Bundle bids permitted, with average of 2.5 items per bundle
  - Multi-unit, with average item quantity of 2.5
  - Free disposal permitted by buyers and sellers
  - Exchange types: 1) Buyer/Seller, 2) Pure bids, 3) Buy&Sell
  - All runs completed in under 3 hours
- **Constraints**
  - Max winners constraint for whole exchange
    - At most 5 of 10 bidders accepted
  - Cost constraint for one bidder
    - First bidder is awarded at least 20% of market by $ value
  - Discount schedule for one bidder
    - Percentage discounts based on $ awarded

Speed of different solution technologies

- **All timing results are for finding an optimal allocation & proving optimality**
- **Solution technologies compared**
  - CPLEX 8.1 out-of-the-box vs. CombineNet's technology
    - Tuned CPLEX is within 10% of CPLEX out-of-the-box
- **Results over all exchange types**

	Avg run time (60 instances)
CPLEX	400 s
CombineNet technology	27 s

Speed by instance type

- **All exchanges, constrained vs unconstrained**

	CONSTRAINED	UNCONSTRAINED
CPLEX	408 s	393 s
CombineNet technology	29 s	24 s

- **All exchanges, different exchange types**

	BUYER/SELLER	PURE	BUY&SELL
CPLEX	349 s	164 s	689 s
CombineNet technology	19 s	14 s	47 s

Factors that affect problem difficulty

In order of impact:

- **Amount of demand for a given item**
  - Higher average bid item quantities make problems much harder
  - Single-unit exchanges are much less complex than multi-unit exchanges
- **Competitiveness of bids**
  - Close bid prices make problem much tougher
    - More possible solutions are close in value
- **Side constraints**
  - May either help or hurt, depending on the problem and constraints
  - Usually hurt, but not relatively as much as in reverse auctions
- **Free disposal**
- **Size of subset of items bidder is interested in**
  - Larger subsets will mean there are more bidders on each item
  - The more bidders on an item, the tougher the problem
- **Buy&Sell bundles**

Conclusions

- **Combinatorial markets of different types have become a reality and CombineNet has a lot of experience designing, building, fielding & hosting them**
- **Combinatorial exchanges are very complex to clear**
  - NP-complete, inapproximable
  - Orders of magnitude more complex than combinatorial auctions or reverse auctions of the same size
- **CombineNet technology is the fastest for the problem by 1-2 orders of magnitude**
- **Optimal clearing scales to reasonable problem sizes**
- **Complexity depends on certain features of the instances, as presented**

Expected FCC exchange model

- **General points**
  - Each license for a frequency range in a region is an item
  - There are # ranges (~35) X # regions (500?) items
- **Aspects that decrease complexity**
  - Each item has a single unit only
  - There is a single seller for each item (though multiple buyers possible)
  - There is a definite structure to bids, by region and frequency range
  - Small sellers and large buyers provide asymmetry
- **Aspects that increase complexity**
  - Substitutability of frequency ranges may explode the size of bids
  - Large bundles are likely for the buyers
  - Potentially several large buyers for each item