APPENDIX C

Clearing Target Optimization

1 Introduction

This appendix provides a final version of Appendix C of the *Auction 1000 Comment PN* setting forth the technical details and mathematical models used for the clearing target optimization. The clearing target optimization determines, for a given clearing target, an assignment of television stations. The clearing target optimization is run as part of the clearing target determination procedure both before the start of the reverse auction bidding process and before any subsequent stage of the auction. This final version of Appendix C is updated to implement the Commission's decisions in the *Auction 1000 Bidding Procedures PN* and its recently concluded coordination agreement with Canada.[1]

As discussed in Section 2 and as illustrated in Figure 1 below, the initial clearing target optimization involves solving a series of optimization problems in order to identify a provisional assignment of television stations to channels that minimizes impairments to forward auction licenses and accomplishes additional objectives. Each step establishes constraints, or limits on any resulting channel assignment, which apply to subsequent steps. Section 3 explains how the clearing target optimization steps used differ between stages of the auction when the clearing target is reduced, and is illustrated in Figure 2.

The first step in the clearing target optimization is to generate constraints that will ensure that every U.S. and Canadian station eligible for protection in the repacking is assigned to either a relinquishment option or a channel in their pre-auction band.[2] This step is described in detail in Sections 2.1

The second step determines additional constraints to assign every Canadian station a channel that satisfies the stipulations within the U.S.–Canada coordination agreement.[3] The steps taken are described in detail in Sections 2.2.

The third step, which applies at the beginning of the auction but not before any subsequent stage of the auction, determines constraints to accommodate the initial bid commitments of stations that are participating in the reverse auction, according to the priorities proposed in the *Comment PN* and adopted in the *Auction 1000 Bidding Procedures PN*.[4] This step is described in detail in Section 2.3.

The next four steps of the clearing target optimization apply the objectives for a channel assignment established in the *Auction 1000 Bidding Procedures PN*. The primary objective is to minimize impaired weighted-pops nationwide, based on the measurement procedure the Commission has adopted. The secondary objective is to maximize the number of weighted Category 1 licenses available in the forward

[1] *Statement of Intent Between the Federal Communications Commission of the United States of America and the Department of Industry Canada Related to the Reconfiguration of Spectrum Use in the UHF Band for Over-the-Air Television Broadcasting and Mobile Broadband Services*, U.S.–Can., Aug. 11, 2015, *available at* https://transition.fcc.gov/ib/sand/agree/files/PASIIC.pdf (*Canadian Coordination*); *see Broadcast Incentive Auction Scheduled to Begin March 29, 2016; Procedures for Competitive Bidding in Auction 1000, Including Initial Clearing Target Determination, Qualifying to Bid, and Bidding in Auctions 1001 (Reverse) and 1002 (Forward)*, GN Docket No. 12-268, AU Docket No. 14-252, 30 FCC Rcd 8975, 8986, para. 16 n.52 (2015) (*Auction 1000 Bidding Procedures PN*).

[2] *See Auction 1000 Bidding Procedures PN*, 30 FCC Rcd at 8984, para. 14 (a feasible channel assignment "satisfies the constraints established in the *Incentive Auction R&O* to make all reasonable efforts to preserve each television station's coverage area and population served").

[3] Under the coordination agreement, full-power Canadian stations may not be assigned to channels in the 600 MHz Band or the additional guard band, with one exception at the 126 MHz clearing target. *See Canadian Coordination*, App. 4, at 13 tbl.4-1 ("Guardband" parameter).

[4] *Auction 1000 Bidding Procedures PN*, 30 FCC Rcd at 8985, para. 14 n.47.

auction. The secondary objective is constrained by the result of the primary objective (the impaired weighted population, rounded up to the nearest integer). The tertiary objective, which is constrained by the first two, is to minimize impaired weighted-pops over all licenses, including licenses with greater than 50 percent of the population subject to impairment. The final objective, which applies only to clearing targets where the lower guard band is 11 MHz, is to minimize the number of stations placed on the lower channel in the lower guard band without changing the stations assigned to channels in the 600 MHz Band. These four steps are described in detail in Sections 2.4–2.7.

2 Initial Clearing Target Optimization

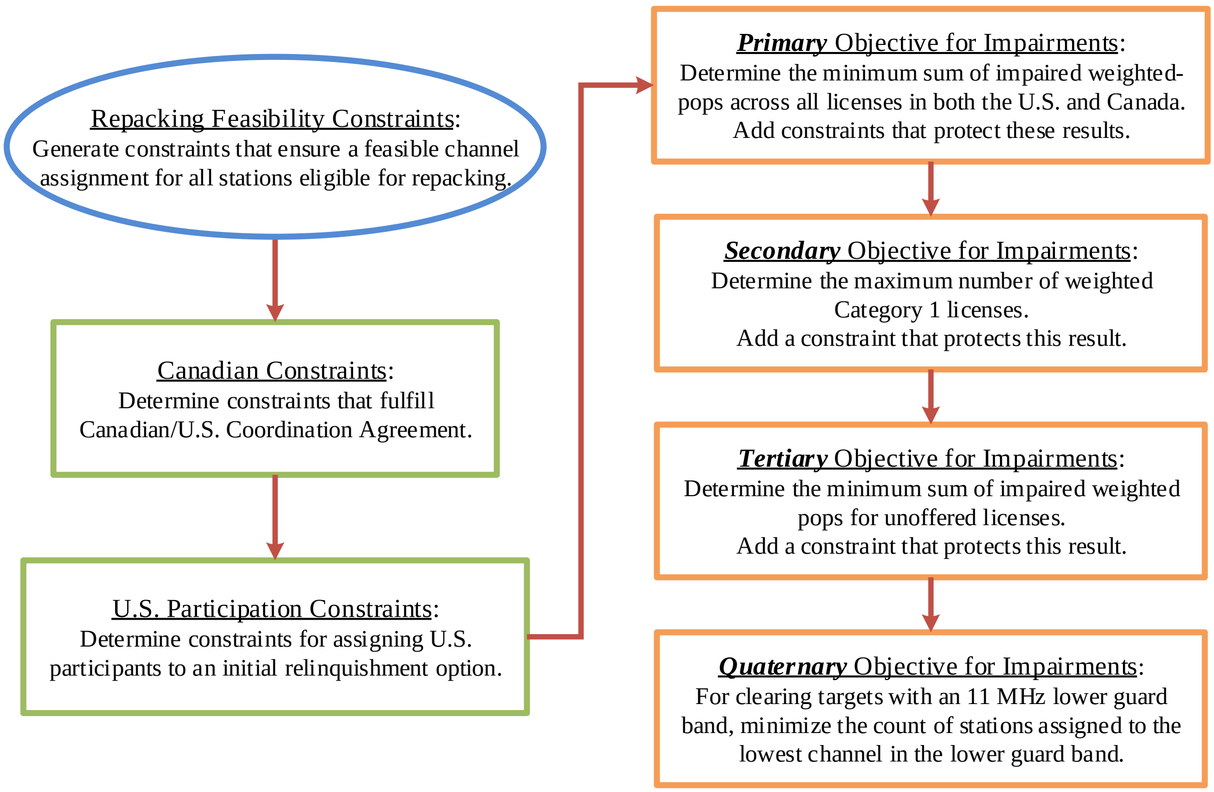

Figure 1: Initial Clearing Target Optimization Flow

2.1.1 Repacking Feasibility Constraints

In the initial clearing target optimization, a feasible assignment is defined as an assignment of TV stations that meets all of the following conditions:

(1) All stations are assigned, either to a channel or to go off-air.
(2) A station can only be assigned to one of its allowable channels as defined in the *domain.csv* file.
(3) A station's assignment must not violate adjacent and co-channel pairwise interference restrictions as defined in the *interference_paired.csv* file.[5]
(4) All non-participating stations are assigned a channel in their pre-auction band, as are stations that are not needed to bid in the auction.
(5) All participating stations in the reverse auction are assigned to an option consistent with the bidder's initial commitment(s) (either to go off-air or to a channel in a band it selected), or to a channel in the bidder's pre-auction band.

[5] *Expanding the Economic and Innovation Opportunities of Spectrum Through Incentive Auctions*, GN Docket No. 12-268, Report & Order, 29 FCC Rcd 6567, 6619, para. 114 (2014) (*Incentive Auction R&O*).

The linear constraints that enforce conditions (1) through (5) are provided below.

Set Definitions:

S is the set of all stations in both Canada and the U.S.

C_s is the set of allowable channels for station s.

For non-participating stations and stations that are not needed, the set C_s consists exclusively of allowable channels in their pre-auction bands, which for UHF stations includes their allowable channels in the 600 MHz Band. For participating stations, the set C_s consists of allowable channels in their pre-auction band as well as channels in the bands associated with their initial relinquishment commitment(s). For participating stations that made an initial commitment to go off-air,[6] the set C_s also consists of channel 0 which indicates an off-air assignment.

Variable Definitions:

$x_{s,c}$ is a binary decision variable which has a value of 1 if station *s* is assigned to channel *c* and 0 otherwise. Note $c = 0$ indicates the option to go off-air.

Explanation of Constraints:

1. **Each station must be assigned.**

$$\sum_{c \in C_s} x_{s,c} = 1 \qquad \forall s \in S$$

This constraint ensures that every station is assigned to exactly one channel from its set of allowable channel assignments.

2. **Station assignments must adhere to the co-channel interference restrictions.**

$$x_{s,c} + x_{s',c} \leq 1 \qquad \forall \{(s,c),(s',c)\} \in CoPairs$$

For every pairwise restriction that precludes two stations from occupying the same channel, a constraint indicates that at most one of the two stations (s and s') can be assigned to that channel c. The set includes all station pairs that cannot occupy the same channel.

3. **Station assignments must adhere to the adjacent channel restrictions.**

$$x_{s,c} + x_{s',c'} \leq 1 \qquad \forall \{(s,c),(s',c')\} \in AdjPairs$$

For every two station-pairs (s,c) and (s',c') where channels c and c' are adjacent and where if station s is on channel c then station s' cannot be on channel c', a constraint allows only one of these two assignments. That is, the constraints enforce the adjacent channel requirements.

4. **The variables can only take on the values zero or one.**

$$x_{(s,c)} \in \{0,1\} \qquad \forall s \in S, \forall c \in C_s$$

For each allowable station-channel combination, the value of the variable $x_{s,c}$ is restricted to be either 0 or 1, *i.e.*, the station is either assigned to the channel or it is not.

As determined by the Commission in the *Auction 1000 Bidding Procedures PN*, no station may be assigned to channels 50 or 51.[7] The clearing target optimization procedure determines a feasible

[6] This set includes stations that initially committed to a VHF option with a back-up option to go off-air.

[7] *See Auction 1000 Bidding Procedures PN*, 30 FCC Rcd at 8999, para. 35.

assignment where all stations can be assigned to some channel other than channels 50 and 51. Once a feasible assignment is found, then channels 50 and 51 are removed from all stations' domains.

2.1.2 Complete Set of Repacking Feasibility Constraints

$\sum_{c\in C_s} x_{s,c} = 1$	$\forall s \in S$	(1)
$x_{s,c} + x_{s',c} \leq 1$	$\forall\{(s,c),(s',c)\} \in CoPairs$	(2)
$x_{s,c} + x_{s',c'} \leq 1$	$\forall\{(s,c),(s',c')\} \in AdjPairs$	(3)
$x_{(s,c)} \in \{0,1\}$	$\forall s \in S, \forall c \in C_s$	(4)

2.2 Optimizations to Satisfy Canadian/US Coordination Agreements on Joint Repacking

This step in the procedure employs five separate optimizations to determine the minimum number of Canadian stations that must be assigned to either the first TV channel adjacent to the guard band between the TV spectrum and the repurposed mobile broadband spectrum (that is, the highest allowable channel in the TV band)[8] or to a channel in the 600 MHz Band.

- (C1): Minimize the count of Canadian full power stations that are assigned to the 600 MHz Band and the highest UHF TV channel
- (C2): Minimize the count of Canadian low-power stations assigned to the 600 MHz Band and the highest UHF TV channel while constraining the count of the high-powered Canadian stations to be no more than that obtained in (C1);
- (C3): If the result of (C1) is greater than zero, maximize the count of the full-power Canadian stations on the highest UHF TV Channel subject to the results obtained in (C1) and (C2). This optimization attempts to assign full-power Canadian stations to the highest UHF TV channel rather than in the 600 MHz Band.
- (C4): If the result of (C2) is greater than zero, maximize the count of the low-power Canadian stations on the highest UHF TV Channel subject to the results obtained in (C1), (C2), and (C3). This optimization attempts to assign low-power Canadian stations to the highest UHF TV channel rather than in the 600 MHz Band.
- (C5): If the result of (C3) is greater than zero, minimize the sum of interference-free population for Canadian full-power stations assigned to the highest UHF TV channel, subject to the results obtained in (C1) through (C4).

2.2.1 (C1): Minimize the Count of Canadian Full-power Stations that are Assigned to the 600 MHz Band and the Highest UHF TV Channel

Subset:

HC denotes the highest UHF TV channel for the given clearing target.

$s \in S_{Can,UHF,FP}$ is the set of full-power UHF-based Canadian stations.

$C_s^{600\cup HC}$ is the set of 600 MHz channels applicable for the giving clearing target as well as the highest UHF TV channel in the clearing target for station s.

[8] Canada requires a larger guard band between TV stations and wireless broadband than does the US. *Canadian Coordination*, App. 4, at 13 tbl.4-1. Thus, Canada considers the highest station in the TV spectrum band (as specified by the US) to be part of their guard band.

Model Formulation for (C1):

$$min\, Z_{C1} = \sum_{s \in S_{Can,UHF,FP}} \sum_{c \in C_s^{600 \cup HC}} x_{s,c}$$

Subject to Constraints:

Repacking Feasibility Constraints (see Section 2.1.2)	(1)-(4)

2.2.2 (C2): Minimize the Count of Canadian Low-power Stations Assigned to the 600 MHz Band and the Highest UHF TV Subject to the Results Obtained in (C1)

Subset:

$S_{Can,UHF,LP}$ is the set of low-power UHF-based Canadian stations.

$C_s^{600MHz \cup HC}$ is the set of 600 MHz channels applicable for the giving clearing target as well as the highest UHF TV channel in the clearing target for station s.

Model Formulation for (C2):

$$min\, Z_{C2} = \sum_{s \in S_{Can,UHF,LP}} \sum_{c \in C_s^{600 \cup HC}} x_{s,c}$$

Subject to:

Repacking Feasibility Constraints (see Section 2.1.2)	(1)-(4)
$\sum_{s \in S_{Can,UHF,FP}} \sum_{c \in C_s^{600 \cup HC}} x_{s,c} \leq Z_{C1}$	(5)

2.2.3 (C3): Maximize the Count of the Full-power Canadian Stations on the Highest UHF TV Channel Subject to the Results Obtained in (C1) and (C2).

This optimization is only performed if the result of (C1) has a value for Z_{C1} that is greater than zero. It attempts to assign full-power stations to the highest UHF TV channel, rather than in the 600 MHz Band, without increasing the number of full-power and low-power Canadian stations in the 600 MHz Band.

Model Formulation for (C3):

$$max\, Z_{C3} = \sum_{s \in S_{Can,UHF,FP}} x_{s,HC}$$

Subject to:

Repacking Feasibility Constraints (see Section 2.1.2)	(1)-(4)
$\sum_{s \in S_{Can,UHF,FP}} \sum_{c \in C_s^{600 \cup HC}} x_{s,c} \leq Z_{C1}$	(5)
$\sum_{s \in S_{Can,UHF,LP}} \sum_{c \in C_s^{600 \cup HC}} x_{s,c} \leq Z_{C2}$	(6)

2.2.4 (C4): Maximize the Count of the Low-power Canadian Stations on the Highest UHF TV Channel Subject to the Results Obtained in (C1), (C2), and (C3).

This optimization is only performed if the result of (C2) has a value for Z_{C2} that is greater than zero. It attempts to assign low-power Canadian stations to the highest UHF TV channel, rather than in the 600 MHz Band, without increasing the numbers of full-power and low-power Canadian stations in the 600 MHz Band, or decreasing the number of full-power Canadian stations assigned to the highest UHF TV channel.

Model Formulation for (C4):

$$max\, Z_{C4} = \sum_{s \in S_{Can,UHF,LP}} x_{s,HC}$$

Subject to:

Repacking Feasibility Constraints (see Section 2.1.2)	(1)-(4)
$\sum_{s \in S_{Can,UHF,FP}} \sum_{c \in C_s^{600 \cup HC}} x_{s,c} \leq Z_{C1}$	(5)
$\sum_{s \in S_{Can,UHF,LP}} \sum_{c \in C_s^{600 \cup HC}} x_{s,c} \leq Z_{C2}$	(6)
$\sum_{s \in S_{Can,UHF,FP}} x_{s,HC} \geq Z_{C3}$	(7)

2.2.5 (C5): Minimize the Interference-free Population of Canadian Full-power Stations Assigned on the Highest UHF TV Channel, Subject to the Results Obtained in (C1)–(C4).

This optimization is only done if (C3) is necessary and if Z_{C3} has a value greater than zero. It attempts to assign the Canadian full-power stations to the to the highest UHF TV channel with the minimum sum of interference-free populations.

Constants

pop_s is the interference-free population of station s.

Model Formulation for (C5):

$$min\, Z_{C5} = \sum_{s \in S_{Can,UHF,FP}} pop_s\, x_{s,HC}$$

Subject to:

Repacking Feasibility Constraints (see Section 2.1.2)	(1)-(4)
$\sum_{s \in S_{Can,UHF,FP}} \sum_{c \in C_s^{600 \cup HC}} x_{s,c} \leq Z_{C1}$	(5)
$\sum_{s \in S_{Can,UHF,LP}} \sum_{c \in C_s^{600 \cup HC}} x_{s,c} \leq Z_{C2}$	(6)
$\sum_{s \in S_{Can,UHF,FP}} x_{s,HC} \geq Z_{C3}$	(7)
$\sum_{s \in S_{Can,UHF,LP}} x_{s,HC} \geq Z_{C4}$	(8)

2.2.6 Complete set of Canadian constraints determined by solving (C1)–(C5)

$$\sum_{s\in S_{Can,UHF,FP}} \sum_{c\in C_s^{600\cup HC}} x_{s,c} \leq Z_{C1} \quad (5)$$

$$\sum_{s\in S_{Can,UHF,LP}} \sum_{c\in C_s^{600\cup HC}} x_{s,c} \leq Z_{C2} \quad (6)$$

$$\sum_{s\in S_{Can,UHF,FP}} x_{s,HC} \geq Z_{C3} \quad (7)$$

$$\sum_{s\in S_{Can,UHF,LP}} x_{s,HC} \geq Z_{C4} \quad (8)$$

$$\sum_{s\in S_{Can,UHF,FP}} pop_s \, x_{s,HC} \leq Z_{C5} \quad (9)$$

Constraint (9) ensures that the sum of interference-free populations of Canadian full-power stations assigned to the highest UHF TV channel is less than or equal to the minimum amount found in (C5).

2.3 Optimizations to Assign US Participating Stations to a Relinquishment Option

Once the Canadian constraints have been determined, the clearing target optimization attempts to ensure that as many US stations as possible will be able to bid in the auction and are initially assigned to the relinquishment option they selected during the initial commitments process. There are four such optimizations. Given the limited capacity of the VHF bands, it may not be possible to assign all participating bidders to their preferred relinquishment option. That is, if more bidders prefer a move to low-VHF or a move to high-VHF than can be accommodated in those bands, then the optimization procedure must initially assign some bidders to an alternative commitment option or their pre-auction band. If some participating stations *only* select a move to a VHF band in their initial commitments and the optimization does not assign them a channel in that VHF band, they will be assigned a channel in their pre-auction band and will not be able to bid in the auction.

- (US1): Determine the minimum number of UHF participating stations that must be assigned to their pre-auction band, subject to the results obtained in (C1) through (C5).
- (US2): Determine the minimum number of VHF participating stations that must be assigned to their pre-auction band, subject to the results obtained in (US1) and (C1) through (C5).
- (US3): Determine the maximum number of participating stations that can be assigned to their preferred relinquishment option, subject to the results obtained in (US1) through (US2) and (C1) through (C5).
- (US4): Determine the maximum number of participating stations that can be assigned to go off-air as an alternative to their preferred relinquishment option, subject to the results obtained in (US1) through (US3) and (C1) through (C5).

Once all four optimizations are completed, the procedure adds the outcomes of (US1) through (US4) and (C1) through (C5) as constraints to the primary, secondary, tertiary, and, if necessary, quaternary clearing target optimizations.

The optimizations outlined above ensure an initial feasible assignment of stations in the event that all participating stations cannot be assigned to their preferred options. The following section provides the mathematical formulations of the optimization models solved in (US1) through (US4) to generate a set of constraints that will be added to the primary clearing target optimization models.

2.3.1 (US1): Minimize the number of UHF stations assigned to their pre-auction band.

In (US1), the optimization seeks a feasible solution that minimizes the number of UHF participating stations that must be assigned to their pre-auction band rather than being given the option of bidding in the auction. The constraints for this first optimization are the feasibility constraints, (1) through (4), plus those needed to satisfy requirements with Canada, (5) through (9). In addition to the variables and sets defined in those sections, subsets of the sets S and C_s are defined here.

Subsets:

S_P is the set of participating U.S. stations.

S_{P_U} is the set of participating U.S. stations whose pre-auction band is UHF.

C_s^H is the set of allowable pre-auction band channels for station s, where station s is a U.S. station.

Model Formulation for (US1):

$$min\, Z_{US1} = \sum_{s \in S_{P_U}} \sum_{c \in C_s^H} x_{s,c}$$

Subject to:

Repacking Feasibility Constraints (see Section 2.1.2)	(1)-(4)
Canadian Constraints (see Section 2.2.6)	(5)-(9)

The objective function minimizes the number Z_{US1} of UHF participating bidders that are assigned to their pre-auction band. Thus, the optimization determines the minimum number of participating stations whose pre-auction band is UHF ($s \in S_{P_U}$) that must be assigned some channel in their pre-auction band ($c \in C_s^H$), considering all stations and their allowable channels and relinquishment options. The value of Z_{US1} will be an integer greater than or equal to zero.

2.3.2 (US2): Minimize VHF stations assigned to their pre-auction band.

(US2) attempts to minimize the number of participating U.S. VHF stations assigned to their respective pre-auction bands, while ensuring that the assignment is feasible and the number of participating UHF stations assigned to their pre-auction band is no more than that found in (US1). The constraints for the optimization in (US2) are those defined in (US1) with the result of (US1) added as an additional constraint. In addition to the variables and sets defined in (US1), an additional subset of the set S is defined here.

Subsets:

S_{P_V} is the set of participating U.S. stations whose pre-auction band is VHF.

Model Formulation for (US2):

$$min\, Z_{US2} = \sum_{s \in S_{P_V}} \sum_{c \in C_s^H} x_{s,c}$$

Subject to:

Repacking Feasibility Constraints (see Section 2.1.2)	(1)-(4)
Canadian Constraints (see Section 2.2.6)	(5)-(9)
$\sum_{s \in S_{P_U}} \sum_{c \in C_s^H} x_{s,c} \leq Z_{US1}$	(10)

Constraint (10) states that the number of the participating U.S. UHF stations assigned to their pre-auction band must be less than or equal to the count obtained in the previous optimization, Z_{US1}.

The objective function minimizes the number Z_{US2} of participating U.S. VHF stations that are assigned to their pre-auction band. Thus, the optimization determines the minimum number of participating U.S.

stations whose pre-auction band is VHF ($s \in S_{P_V}$) that must be assigned a channel in their pre-auction band ($c \in C_s^H$), considering all stations and their allowable channels and relinquishment options and the minimum number Z_{US1} of UHF stations that must be assigned to their pre-auction band. The value of Z_{US2} will be an integer number greater than or equal to zero.

2.3.3 (US3): Maximize the number of stations assigned to their preferred relinquishment option

(US3) attempts to maximize the number of participating U.S. stations that are assigned to their preferred relinquishment option, while ensuring that the assignment is feasible and the number of stations assigned to their pre-auction band is no more than the minimums found in (US1) and (US2). An additional subset of the set C_s is defined here.

Subset:

C_s^{pref} is the set of allowable channels for a participating U.S. station s in its preferred option.

Note: For stations whose preferred option is to go off-air, the set C_s^{pref} consists solely of 0.

Model Formulation for (US3):

$$max\, Z_{\text{US3}} = \sum_{s \in S_P} \sum_{c \in C_s^{pref}} x_{s,c}$$

Subject to:

Repacking Feasibility Constraints (see Section 2.1.2)	(1)-(4)
Canadian Constraints (see Section 2.2.6)	(5)-(9)
$\sum_{s \in S_{P_U}} \sum_{c \in C_s^H} x_{s,c} \leq Z_{US1}$	(10)
$\sum_{s \in S_{P_V}} \sum_{c \in C_s^H} x_{s,c} \leq Z_{US2}$	(11)

Constraint (11) states that the sum of the VHF participating U.S. stations assigned to their pre-auction band must be less than or equal to the result of the second optimization, namely the value Z_{US2}.

The objective function maximizes the total number Z_{US3} of participating stations that are assigned to their preferred option. The value of Z_{US3} will be an integer greater than or equal to zero.

2.3.4 (US4): Maximize the number of stations assigned to their option of going off the air

If it is not possible to assign all participating stations to their preferred relinquishment option, (US4) seeks to maximize the number of participating stations assigned to going off the air as an alternative option. Being initially assigned the option to go off the air, rather than being assigned to another band, will ensure that as many of the stations assigned to an alternative option as possible have the flexibility to move to bid for their other relinquishment options during the reverse auction.

The optimization model solved in (US4) determines a feasible assignment and assigns as many participating stations as possible to go off the air given the constraints that (a) there cannot be more than Z_{US1} UHF participating stations assigned to channels in the UHF band, (b) there cannot be more than Z_{US2} VHF participating stations assigned to pre-auction band channels in the VHF band, and (c) at least Z_{US3} participating stations are assigned to their preferred option.

Model Formulation for (US4):

$$\max Z_{US4} = \sum_{s \in S_P} x_{s,0}$$

Subject to:

Repacking Feasibility Constraints (see Section 2.1.2)	(1)-(4)
Canadian Constraints (see Section 2.2.6)	(5)-(9)
$\sum_{s \in S_{P_U}} \sum_{c \in C_s^H} x_{s,c} \leq Z_{US1}$	(10)
$\sum_{s \in S_{P_V}} \sum_{c \in C_s^H} x_{s,c} \leq Z_{US2}$	(11)
$\sum_{s \in S_P} \sum_{c \in C_s^{pref}} x_{s,c} \geq Z_{US3}$	(12)

Constraint (12) requires that the number of participating stations assigned to their preferred option must be greater than or equal to the result of the third optimization, namely the value Z_{US3}.

The objective function maximizes the number Z_{US4} of participating stations that are assigned to go off the air. The value of Z_{US4} will be an integer greater than or equal to zero.

2.3.5 Complete set of U.S. Participation Constraints as determined by solving (US1)-(US4)

$\sum_{s \in S_{P_U}} \sum_{c \in C_s^H} x_{s,c} \leq Z_{US1}$	(10)
$\sum_{s \in S_{P_V}} \sum_{c \in C_s^H} x_{s,c} \leq Z_{US2}$	(11)
$\sum_{s \in S_P} \sum_{c \in C_s^{pref}} x_{s,c} \geq Z_{US3}$	(12)
$\sum_{s \in S_P} x_{s,0} \geq Z_{US4}$	(13)

Constraint (13) requires that the number of participating stations assigned to their alternative option to go off-air must be greater than or equal to the result of the fourth optimization, namely the value Z_{US4}.

2.4 Primary Clearing Target Optimization

The next step in determining an initial assignment of participating and non-participating stations is to solve the primary clearing target optimization, which minimizes the impact of impairing TV stations to forward auction licenses, given a specified clearing target and subject to feasibility constraints and the results of (C1) through (C5) and (US1) through (US4). Thus, constraints (1) through (13) above are included in all of the subsequent optimizations. The primary clearing target optimization model seeks a feasible assignment of stations such that the sum of impaired weighted-pops across all licenses in the 600 MHz Band is minimized.

Because of the recently signed *Canadian Coordination Agreements*, there are two components to this primary optimization procedure. (P1) minimizes the maximum country-specific impairment percentage. Thus, the optimization calculates the sum of impaired weighted-pops across all licenses in that country divided by the population of the country. This optimization tries to push the maximum percentage interference of *each* country to be below the near-nationwide standard set for that clearing target.[9] (P2) then minimizes the sum of impaired weighted-pops across all licenses in both countries, while making

[9] *Auction 1000 Bidding Procedures PN*, 30 FCC Rcd at 9001, para. 39 fig.2 (listing the near-nationwide impairment threshold for each clearing target).

sure that neither country has a weighted-pop interference percentage that is greater than the impairment percentage computed in (P1) or the near-nationwide standard, whichever is greater. The results of these two optimizations work to ensure that each country's impairment level is below the near-nationwide standard set for the clearing target and that total impaired weighted-pops are minimized.

It is possible that if the results of the primary clearing target optimizations indicate that one or both countries do not satisfy the near-nationwide threshold needed for that clearing target, the threshold may be satisfied when using the more precise 2km x 2km grid calculation (rather than the county-aggregated methodology). For this reason, regardless of the results from the primary optimization, the secondary, tertiary and, where appropriate, quaternary optimizations are performed. The final resulting assignment of stations to the 600 MHz Band then uses the *TVStudy* data to do a careful evaluation of impairments at the 2km x 2km level. These calculations determine if the clearing target threshold has been met.[10] If the result of the more careful calculations do not satisfy the near-nationwide threshold for impairments, that clearing target will not be selected.

In addition to the constraints derived above and denoted (1) through (13), the primary clearing target optimization also uses the set of ISIX constraints, denoted (14) through (20) below, that determine the impairment created by assigning any station to the 600 MHz Band.[11] For an assignment of stations to channels, these constraints determine the percent of population considered impaired for each license, and are constructed in such a way as to avoid double counting of population where the contours of assigned TV stations overlap with each other. The ISIX constraints also ensure that a license that is more than 50 percent impaired is considered 100 percent impaired, and so will not be offered in the clock phase of the forward auction.[12]

The following are the formulations for both components of the primary clearing target optimization.

2.4.1 (P1): Minimize the maximum percentage weighted impairment incurred in each country.

(P1) minimizes the maximum of the percentage impairments incurred in the U.S. and in Canada.

Variables

$y_{a,l}$ is a decision variable which has a value of 1 if county-tile a is impaired for license l and 0 otherwise.

ρ_l is the percentage of population in license l with predicted impairment.

N_l is a binary variable which has a value of 1 if the license is more than 50 percent impaired.

$MaxImpairment$ is the maximum of the impairment percentages incurred in the U.S. and Canada.

Set Definitions

A_l is the set of county-tiles a covered by license l which can be impaired partially or fully by at least one (facility, channel) pair.

$K = \{US, CA\}$ is the set of countries.

L is the set of licenses considered for the given clearing target; each license is defined by a clearing target, market id, and block. L contains all licenses in both the US and in Canada, since any of these licenses has the potential to be impaired.

L_k is the set of licenses for country $k \in K$.

[10] *See Auction 1000 Bidding Procedures PN*, 30 FCC Rcd at 8985–86, para. 16, 8988–89, para. 21.

[11] For more details on the ISIX constraints, *see* App. B.

[12] *See Auction 1000 Bidding Procedures PN*, 30 FCC Rcd at 9000, para. 38 n.142.

$SC_{a,l}$ is the set of impairing (facility, channel) pairs which impair county-tile a in license l.

Constants

i_l is the weighting associated with license l.

w_l is the weighted-pops associated with license l, which is equal to i_l multiplied by the population associated with license l.

$pct_{a,l}$ is the percent of license l's population in county-tile a.

Model Formulation for (P1):

$$Z_{P1} = min\, MaxImpairment$$

Subject to:

Repacking Feasibility Constraints (see Section 2.1.2)		(1)-(4)
Canadian Constraints (see Section 2.2.6)		(5)-(9)
U.S. Participation Constraints (see Section 2.3.5)		(10)-(13)
$\sum_{a \in A_l} pct_{a,l}\, y_{a,l} = \rho_l$	$\forall l \in L$	(14)
$x_{s,c} \leq y_{a,l}$	$\forall (s,c) \in SC_{a,l}, a \in A_l, l \in L$	(15)
$0 \leq y_{a,l} \leq 1$	$\forall a \in A_l, l \in L$	(16)
$0 \leq \rho_l \leq 1$	$\forall l \in L$	(17)
$\rho_l \leq .5 + (.5)N_l$	$\forall l \in L$	(18)
$\rho_l \geq N_l$	$\forall l \in L$	(19)
$N_l \in \{0,1\}$	$\forall l \in L$	(20)
$MaxImpairment \geq \sum_{l \in L_k} w_l\, \rho_l / \sum_{l \in L_k} w_l$	$\forall k \in K$	(21)

Explanation of New Constraints:

14: Calculation of the total percent of population impaired in each license

$$\sum_{a \in A_l} pct_{a,l}\, y_{a,l} = \rho_l \qquad \forall l \in L$$

For every license, this constraint calculates the total percent of population impaired in the license by summing the percent of population impaired with respect to the market for all the county-tiles a that have broadband service respectively in license l.

15: Constraints that set the county variables to 1

$$x_{s,c} \leq y_{a,l} \qquad \forall (s,c) \in SC_{a,l}, a \in A_l, l \in L$$

For each county in each license, these constraints set the county variable to 1 when a specific (facility, channel) assignment creates impairment. Note that the value of $y_{a,l}$ remains 1 even if multiple channel assignments force the county to be impaired.

16: Constraints that restrict the value of the county variables

$0 \le y_{a,l} \le 1$ $\quad\forall a \in A_l, l \in L$

For each county in each license, these constraints restrict the value of the county variables to be between 0 and 1 inclusive. Note that the constraints are constructed in such a way that, when combined with the objective to minimize the sum of impaired weighted-pops (see Appendix B), these variables will take on the value 0 when the county is not impaired, or 1 when it is. As a result, the model can consider the variables to be continuous but in practice they will be binary.

17: Constraints that restrict the values of the total percent of population with predicted impairment variables

$0 \le \rho_l \le 1$ $\quad\forall l \in L$

For each license l, these constraints restrict the value of the total percent of population with predicted impairment variables to be between 0 and 1 inclusive. A solution value of 0 indicates that there is no predicted impairment in license l, while a value of 1 indicates that this license is predicted to be 100 percent impaired.

18: Constraints that set binary variable N_l to 1

$\rho_l \le .5 + .5N_l$ $\quad\forall l \in L$

For each license l, this constraint will force the variable N_l to be 1 whenever the calculated value of ρ_l is greater than or equal to 50 percent.

19: Constraints that set the percentage of impairment, ρ_l, to 1

$\rho_l \ge N_l$ $\quad\forall l \in L$

For each license l, this constraint will set the impairment percentage of license l to be 100 percent whenever the variable N_l is set to 1. This constraint is coupled with constraint (18) to force the total impairment of the license. Thus, whenever the population impairment percentage is greater than 50 percent, the license is considered completely impaired since it will not be available in the forward auction.

20: Constraints that restrict the value of variable N_l

$N_l \in \{0,1\}$ $\quad\forall l \in L$

For each license l, the variable N_l is restricted to the value 0 or 1.

21: Constraints that find the maximum impairment between the two countries

$MaxImpairment \ge \sum_{l \in L_k} w_l \, \rho_l / \sum_{l \in L_k} w_l$ $\quad\forall k \in K$

For each country k, $MaxImpairment$ must be greater than the calculated percentage impairment of each country.

2.4.2 (P2): Minimize the total amount of impaired weighted population in the U.S. and Canada.

The second component of the primary clearing target optimization minimizes the amount of impairment incurred by both countries, while maintaining that the maximum percentage of impairment in each country cannot be increased above the result of P1 or the clearing target's threshold, whichever is larger. Thus, the first minimizes the maximum impairment each country can incur and this second optimization ensures that the total weighted-pops impaired is minimized.

Constants

$threshold_{CT}$ is the threshold for a specific clearing target CT.

Model Formulation for (P2):

$$Z_{P2} = min \sum_{l \in L} w_l \, \rho_l$$

Subject to:

Repacking Feasibility Constraints (see Section 2.1.2)		(1)-(4)
Canadian Constraints (see Section 2.2.6)		(5)-(9)
U.S. Participation Constraints (see Section 2.3.5)		(10)-(13)
$\sum_{a \in A_l} pct_{a,l} \, y_{a,l} = \rho_l$	$\forall l \in L$	(14)
$x_{s,c} \le y_{a,l}$	$\forall (s,c) \in SC_{a,l}, a \in A_l, l \in L$	(15)
$0 \le y_{a,l} \le 1$	$\forall a \in A_l, l \in L$	(16)
$0 \le \rho_l \le 1$	$\forall l \in L$	(17)
$\rho_l \le .5 + (.5) N_l$	$\forall l \in L$	(18)
$\rho_l \ge N_l$	$\forall l \in L$	(19)
$N_l \in \{0,1\}$	$\forall l \in L$	(20)
$\sum_{l \in L_k} w_l \, \rho_l / \sum_{l \in L_k} w_l \le max\{Z_{P1}, threshold_{CT}\}$	$\forall k \in K$	(21)

Explanation of New Constraint:

Constraint (21) limits the impairment percentage each country can incur to either the result of (P2) or the clearing target threshold whichever is larger.

2.4.3 Complete set of constraints associated with the Primary Clearing Target

$\sum_{a \in A_l} pct_{a,l} \, y_{a,l} = \rho_l$	$\forall l \in L$	(14)
$x_{s,c} \le y_{a,l}$	$\forall (s,c) \in SC_{a,l}, a \in A_l, l \in L$	(15)
$0 \le y_{a,l} \le 1$	$\forall a \in A_l, l \in L$	(16)
$0 \le \rho_l \le 1$	$\forall l \in L$	(17)
$\rho_l \le .5 + (.5) N_l$	$\forall l \in L$	(18)
$\rho_l \ge N_l$	$\forall l \in L$	(19)
$N_l \in \{0,1\}$	$\forall l \in L$	(20)
$\sum_{l \in L_k} w_l \, \rho_l / \sum_{l \in L_k} w_l \le max\{Z_{P1}, threshold_{CT}\}$	$\forall k \in K$	(21)
$\sum_{l \in L} w_l \, \rho_l \le Z_{P2}$		(22)

Constraint (22) states that the percent total impaired weighted pops of any assignment must be less than or equal to the result obtained from (P2) of the primary clearing target optimization rounded up to the nearest integer.

2.5 Secondary Clearing Target Optimization

The next step is to determine, as a secondary objective, the maximum weighted number of Category 1 licenses in Canada and the US given that the maximum impairment cannot be greater than that determined by the primary clearing target optimization. Thus, the secondary objective will function primarily as a tie-breaker in choosing a provisional TV channel assignment plan: when more than one potential plan exists with the same minimum level of impairment as that identified by the primary objective optimization, the secondary objective will seek one that maximizes the weighted number of Category 1 licenses.

The following is the formulation of the secondary clearing target optimization.

Variables

$G_{1,l}$ is a binary variable which has a value of 1 if the licenses can be categorized as a Category 1 license based on the calculated impairment, and 0 otherwise.

Model Formulation for the Secondary Clearing Target Optimization:

$$Z_{Secondary} = max \sum_{l \in L} i_l G_{1,l}$$

Subject to:

Repacking Feasibility Constraints (see Section 2.1.2)		(1)-(4)
Canadian Constraints (see Section 2.2.6)		(5)-(9)
U.S. Participation Constraints (see Section 2.3.5)		(10)-(13)
Primary Clearing Target Constraints (see Section 2.4.3)		(14)-(22)
$\rho_l \leq .15 + .85(1 - G_{1,l})$	$\forall l \in L$	(23)
$G_{1,l} \in \{0,1\}$	$\forall l \in L$	(24)

Explanation of New Constraints:

23: Set $G_{1,l}$ to one when license l is a Category 1 license

$\rho_l \leq .15 + .85(1 - G_{1,l}) \qquad \forall l \in L$

Constraints (23) forces variable $G_{1,l}$ to be zero if it is not a Category 1 license.

24: $G_{1,l}$ must be binary

$G_{1,l} \in \{0,1\} \qquad \forall l \in L$

For each license l, the variable $G_{1,l}$ is restricted to the value 0 or 1.

2.5.1 Complete set of constraints associated with the Secondary Clearing Target

$\rho_l \leq .15 + .85(1 - G_{1,l})$	$\forall l \in L$	(23)
$G_{1,l} \in \{0,1\}$	$\forall l \in L$	(24)
$\sum_{l \in L} i_l G_{1,l} \geq Z_{Secondary}$		(25)
$\sum_{l \in L} i_l (1 - G_{1,l} - N_l) \geq TotG_2$		(26)

Constraint (25) states that the total number of weighted Category 1 licenses may not be less than then the number found in the secondary clearing target optimization. $Z_{Secondary}$ will be rounded down to the nearest integer.

Constraint (26) states that the total number of weighted Category 2 licenses may not be less than then the number found in the result of the secondary clearing target optimization ($TotG_2$).

2.6 Tertiary Clearing Target Optimization

The provisional TV channel assignment plan determined by applying the first two objectives may include licenses that cannot be categorized as either Category 1 or Category 2 because more than 50 percent of the population is subject to impairment. The optimization procedure will apply a tertiary objective in order to maximize the potential value in a subsequent spectrum auction of these more heavily impaired licenses. More specifically, the tertiary objective will seek to minimize impaired weighted-pops over all licenses, including licenses with more than 50 percent of the population subject to impairment.[13] The tertiary objective will be constrained by the first two objectives: it will be applied only to the extent that it neither increases the impaired weighted pops resulting from the primary optimization nor reduces the weighted number of Category 1 licenses resulting from the secondary optimization. The combined impairment percentage is rounded up to the nearest integer and the weighted number of Category 1 licenses is rounded down to the nearest integer. Further, applying the tertiary objective will not decrease the weighted number of Category 2 licenses found by applying the primary and secondary objectives.

The following is the formulation of the tertiary clearing target optimization.

Variables

ρ'_l is the percentage of population in license l with predicted impairment, while not counting licenses with predicted impairment above 50 percent as being 100 percent impaired.

Model Formulation for the Tertiary Clearing Target Optimization:

$$Z_{Tertiary} = min \sum_{l \in L} w_l \, \rho'_l$$

[13] The primary and secondary objectives will count any license with greater than 50 percent impaired weighted-pops as 100 percent impaired.

Subject to:

Repacking Feasibility Constraints (see Section 2.1.2)		(1)-(4)
Canadian Constraints (see Section 2.2.6)		(5)-(9)
U.S. Participation Constraints (see Section 2.3.5)		(10)-(13)
$\sum_{a\in A_l} pct_{a,l}\, y_{a,l} \leq \rho_l$ (note this change in sense)	$\forall l \in L$	(14)
Primary Clearing Target Constraints (see Section 2.4.3)		(15)-(22)
Secondary Clearing Target Constraints (see Section 2.5.1)		(23)-(26)
$\sum_{a\in A_l} pct_{a,l}\, y_{a,l} \leq \rho'_l$	$\forall l \in L$	(27)
$0 \leq \rho'_l \leq 1$	$\forall l \in L$	(28)

Explanation of New Constraints:

27: Calculation of the total percent of population impaired in each license

$\sum_{a\in A_l} pct_{a,l}\, y_{a,l} \leq \rho'_l$ $\forall l \in L$

For every license, Constraint (27) sets the lower bound on the total percent of population impaired in the license without counting licenses not offered in the forward auction as 100 percent impaired, by summing the percent of population impaired with respect to the market for all the county-tiles a that have broadband service respectively in license l.

28: Constraints that restrict the values of the total percent of population with predicted impairment variables

$0 \leq \rho'_l \leq 1$ $\forall l \in L$

For each license l, these constraints restrict the value of the total percent of population with predicted impairment without counting licenses not offered in the forward auction as 100 percent impaired variables to be between 0 and 1 inclusive. A solution value of 0 indicates that there is no predicted impairment in license l, while a value of 1 indicates that this license is predicted to be 100 percent impaired.

2.6.1 Complete set of constraints associated with the Tertiary Clearing Target

$\sum_{a\in A_l} pct_{a,l}\, y_{a,l} \leq \rho'_l$	$\forall l \in L$	(27)
$0 \leq \rho'_l \leq 1$	$\forall l \in L$	(28)
$\sum_{l\in L} w_l\, \rho'_l \leq Z_{Tertiary}$		(29)

Constraint (29) ensures that the maximum potential value of licenses not offered in the forward auction is maintained for a subsequent spectrum auction.

2.7 Quaternary Clearing Target Optimization

For clearing targets with an 11 MHz lower guard band,[14] the final step in determining an initial assignment of participating and non-participating stations is to minimize the count of stations assigned to the lowest channel in the lower guard band. We define $HC + 1$ to be the lowest channel above the highest allowable channel in the TV band for the given clearing target.

In this optimization, all stations assigned in the 600 MHz Band above $HC + 1$ will remain on their assigned channel, and no additional stations will be assigned to the 600 MHz Band. This will be accomplished by removing the appropriate channels from each station's domain. Therefore, there is no need to include the constraints associated with the primary, secondary, and tertiary optimizations.

The following is the full formulation of the quaternary clearing target optimization.

Model Formulation for the Quaternary Clearing Target Optimization:

$$min\, Z_{Quaternary} = \sum_{s \in S} x_{s,HC+1}$$

Subject to:

Repacking Feasibility Constraints (see Section 2.1.2)	(1)-(4)
Canadian Constraints (see Section 2.2.6)	(5)-(9)
U.S. Participation Constraints (see Section 2.3.5)	(10)-(13)

3 Clearing Target Optimization during the Auction

The Clearing Target Optimization will also be used between stages in order to determine an assignment of stations consistent with the lower clearing target, which will establish forward auction license impairments for the next stage. In this section, the optimization models and their mathematical formulations are discussed.

The Clearing Target Optimization will be run between stages to account for the additional UHF channel or channels available in the television portion of the band, which will impact any channel assignments that must be made in the 600 MHz Band. Thus, before the start of a new stage, the Clearing Target Optimization software will re-shuffle the UHF band based on the new clearing target and incorporating the feasibility, Canadian Coordination, and ISIX constraints into the primary through quaternary optimizations aimed at minimizing impaired weighted-pops when assigning stations to channels in the 600 MHz Band. The reverse auction bidding system will then use the new assignment of stations as the initial assignment of UHF stations for the next stage, fixing the assignment of those stations assigned in the 600 MHz Band and keeping tentative those assigned to be repacked into the television portion of the UHF band. The forward auction will use the corresponding impairments for licenses offered in the next stage of the forward auction. A flow chart of the clearing target optimization used between stages is shown in Figure 2 below:

[14] *See Auction 1000 Bidding Procedures PN*, 30 FCC Rcd at 8981, para. 5 fig.1.

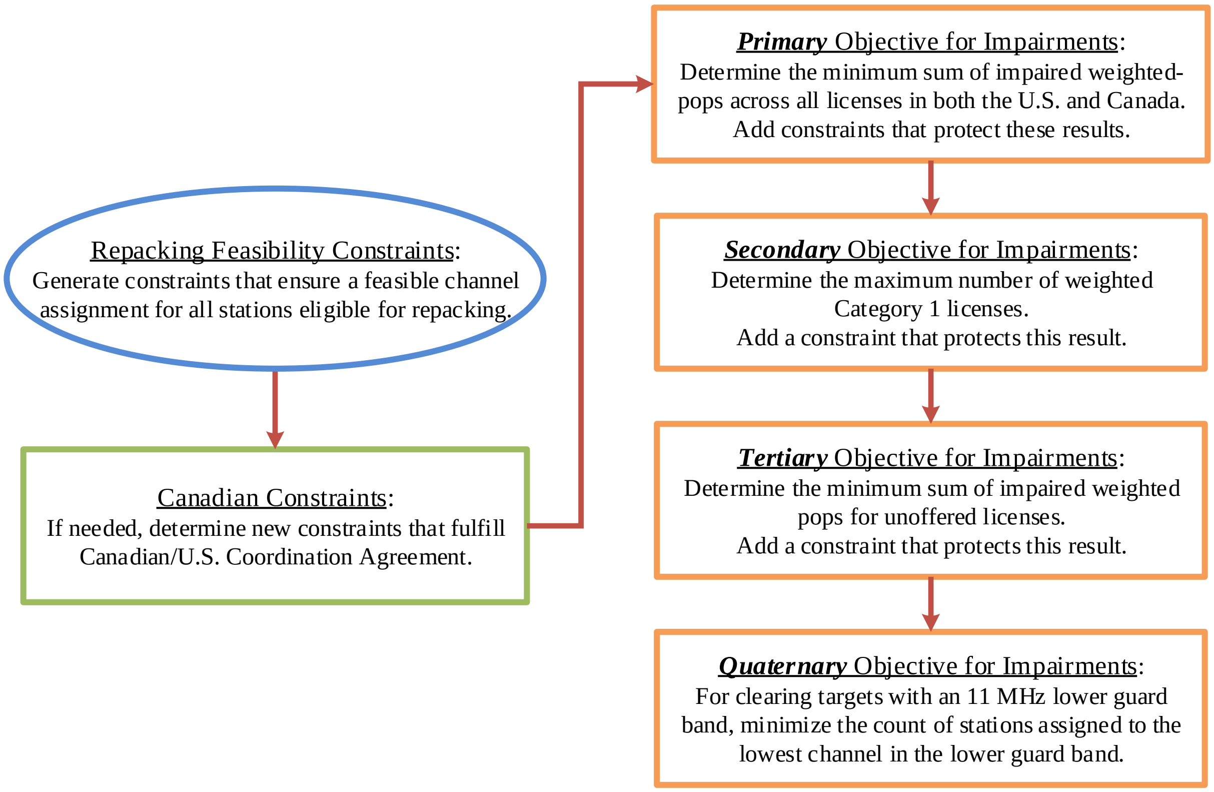

Figure 2: Between Stages Clearing Target Optimization Flow

Model Formulation

The formulation of the optimization model is the same as it is in the initial clearing target optimization with the one exception that the optimizations associated with determining constraints for the participating U.S. stations are not solved. When performing the clearing target optimization between stages, the objective is to "reshuffle" the assignment of stations to channels in the UHF band so as to minimize impaired weighted pops in the 600 MHz Band. Specifically, the clearing target optimization solved in between stages differs from the initial clearing target optimization in the following ways:

(1) The set of stations S is reduced to stations assigned to the UHF band at the completion of the previous stage.
(2) (US1) through (US4) do not need to be performed because all participating stations are already assigned to a relinquishment option.
(3) (C1) through (C5) must be redone for each clearing target until it is established that all Canadian stations can be assigned to the TV band. Once it is established that $Z_{C1} = Z_{C2} = 0$ for a given clearing target, then (C1) through (C5) will not need to be performed for any lower clearing target. The domain allowed for the Canadian station will be set to include only channels below the highest UHF channel within the clearing target.
(4) The primary, secondary and tertiary, and sometimes quaternary optimizations are performed in order to
   - reduce the total impaired weighted-pops,
   - encourage the shifting of Category 2 licenses to Category 1 licenses,
   - make available in the forward auction those licenses that were not previously, and
   - make the licenses not available in the forward auction more valuable in a future auction.